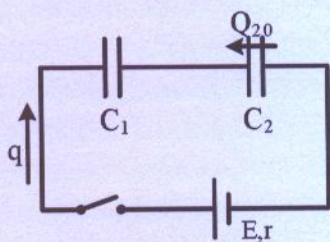


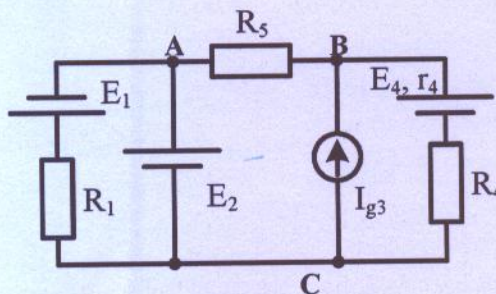
Elektrotehnika

pismeni ispit, 17.8.2025.

1. U kolu na Slici 1 prekidač je otvoren, kondenzator C_2 je opterećen početnom količinom naelektrisanja $Q_{20} = 3 \mu\text{C}$, a kondenzator C_1 je neopterećen. Odrediti naelektrisanje q koje će proteći nakon zatvaranja prekidača i elektrostatičke energije kondenzatora C_1 i C_2 . Poznati su parametri: $E = 5 \text{ V}$, $r = 5 \Omega$, $C_1 = 6 \mu\text{F}$ i $C_2 = 3 \mu\text{F}$. (8 poena)



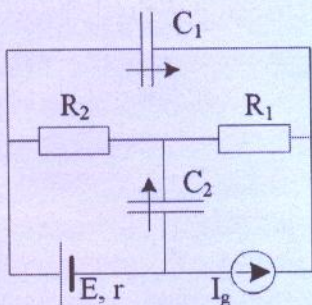
Slika 1



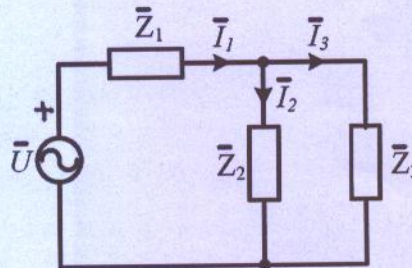
Slika 2

2. U kolu na Slici 2 poznate su vrednosti parametara elemenata: $R_1 = R_4 = R_5 = 10 \Omega$, $E_1 = 20 \text{ V}$, $E_2 = 10 \text{ V}$, $E_4 = 25 \text{ V}$, $r_4 = 5 \Omega$ i $I_{g3} = 1 \text{ A}$. Primenom Tevenenove teoreme odrediti intenzitet struje realnog naponskog generatora $E_4(r_4)$, a nakon toga i njegovu snagu. (12 poena)

3. Na Slici 3 poznato je $R_1 = R_2 = 10 \text{ k}\Omega$, $E = 20 \text{ V}$, $r = 5 \text{ k}\Omega$, $I_g = 10 \text{ mA}$ i $C_1 = C_2 = 22 \text{ nF}$. Odrediti količine naelektrisanja na kondenzatorima (u skladu sa zadatim referentnim smerom). (8 poena)



Slika 3



Slika 4

4. Na Slici 4 je prikazano kolo naizmenične struje koje čine naponski generator $u(t) = 100 \cos(\omega t) \text{ V}$ i tri potrošača poznatih vrednosti kompleksnih impedansi:

$$\bar{Z}_1 = j10 \Omega, \bar{Z}_2 = 10 \Omega \text{ i } \bar{Z}_3 = -j10 \Omega.$$

a) Odrediti kompleksne izraze za struje u svim granama kola. (6 poena)

b) Odrediti trenutnu vrednost napona na impedansi Z_1 . (2 poena)

c) Odrediti aktivnu, reaktivnu i prividnu snagu paralelne veze Z_2 i Z_3 . (4 poena)

5. Na Slici 5 je prikazan trofazni sistem generator

- potrošač. Ems generatora iznose:

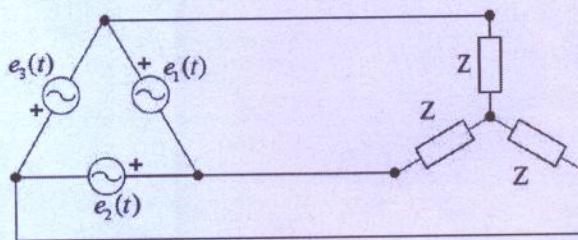
$$e_1(t) = 400 \cdot \cos(314t - \pi/2) \text{ V}$$

$$e_2(t) = 400 \cdot \cos(314t + \pi/6) \text{ V}$$

$$e_3(t) = 400 \cdot \cos(314t + 5\pi/6) \text{ V},$$

dok impedansa jedne faze potrošača iznosi

$$\bar{Z} = 20 + j10 \Omega.$$

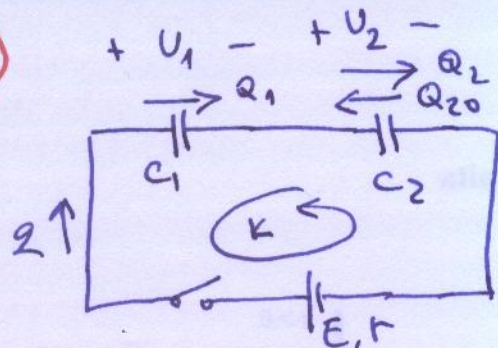


Slika 5

a) Nacrtati fazorski dijagram linijskih napona. (3 poena)

b) Odrediti efektivnu vrednost linijskih struja, faktor snage i reaktivnu snagu potrošača. (7 poena)

①



$$K: -E + U_2 + U_1 = 0$$

$$U_1 + U_2 = E$$

$$\frac{Q_1}{C_1} + \frac{Q_2}{C_2} = E$$

$$\frac{Q}{C_1} + \frac{Q - Q_{20}}{C_2} = E$$

$$Q \left(\frac{1}{C_1} + \frac{1}{C_2} \right) = E + \frac{Q_{20}}{C_2}$$

$$Q = \frac{E + Q_{20}/C_2}{\frac{1}{C_1} + \frac{1}{C_2}} = \frac{5 + 1}{\frac{1}{6\mu} + \frac{1}{3\mu}}$$

$$Q = 12 \mu C$$

redne vezg:

$$Q_1 = Q = 12 \mu C$$

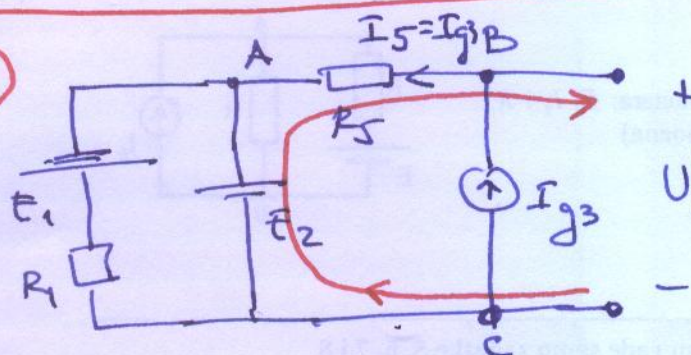
$$Q_2 = Q - Q_{20} = 9 \mu C$$

↑
suprotan smer

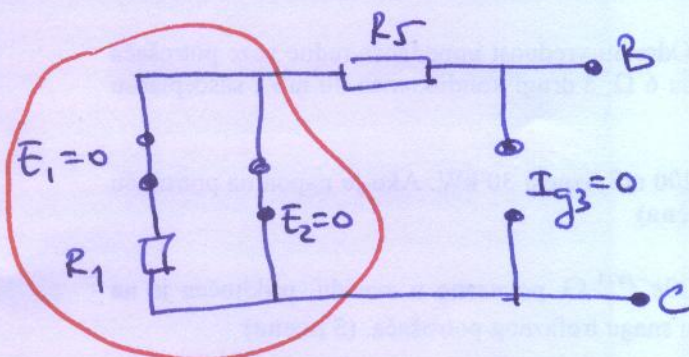
$$W_1 = \frac{1}{2} Q_1^2 / C_1 = 12 \mu J$$

$$W_2 = \frac{1}{2} Q_2^2 / C_2 = 13,5 \mu J$$

②

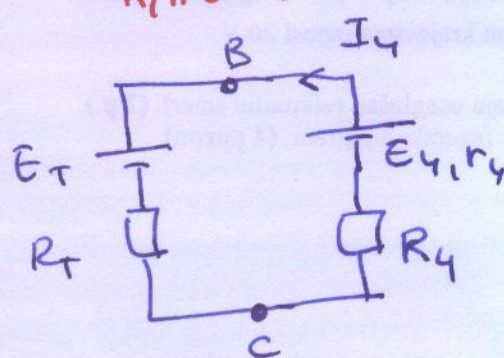


$$U_{BC}^{ov} = E_r = E_2 + R_5 \cdot I_{g3} = 10 + 10 \cdot 1 = 20V$$



$$R_{BC}^{ov} = R_r = R_5 + (R_1 \parallel 0) = R_5 = 10 \Omega$$

$$R_1 \parallel 0 = 0$$

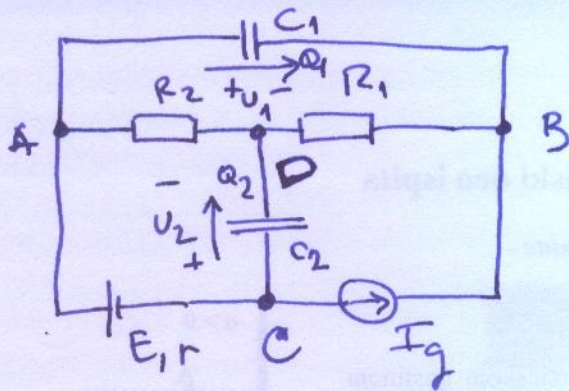


$$I_4 = \frac{E_4 - E_T}{R_T + R_4 + r_4} = \frac{25 - 20}{10 + 10 + 5} = \frac{5}{25} = 0,2 A$$

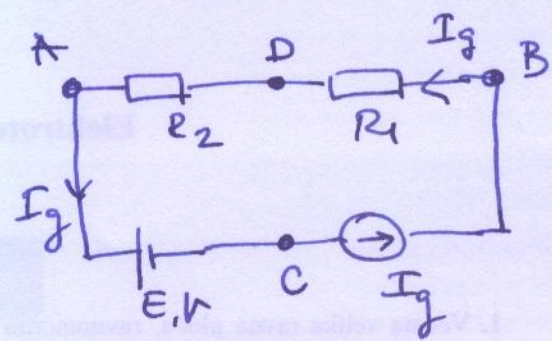
$$P_{E_4, r_4} = E_4 I_4 - r_4 I_4^2 = 25 \cdot 0,2 - 5 \cdot (0,2)^2$$

$$P_{E_4, r_4} = 5 - \frac{1}{5} = 4,8 W$$

3



$$\Rightarrow I_{C1} = 0 \\ I_{C2} = 0$$



$$Q_1 = C_1 U_1 = 22 \mu \cdot (-200)$$

$$Q_1 = -4.4 \mu C$$

$$Q_2 = C_2 U_2 = 22 \mu \cdot (-170)$$

$$Q_2 = -3.74 \mu C$$

$$U_1 = U_{AB} = -(R_1 + R_2) I_g$$

$$U_1 = -20k \cdot 10m = -200V$$

$$U_2 = U_{CD} = -r I_g - E - R_2 I_g$$

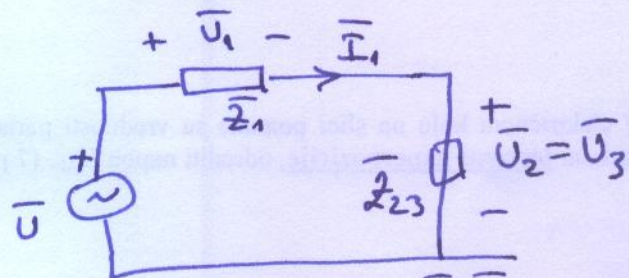
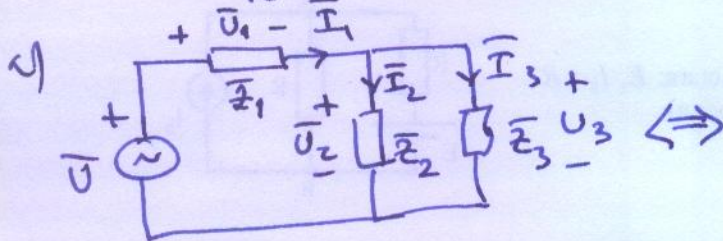
$$U_2 = -5k \cdot 10m - 20 - 10k \cdot 10m$$

$$U_2 = -50 - 20 - 100$$

$$U_2 = -170V$$

4

$$\bar{U} = \frac{100}{\sqrt{2}} e^{j0} = 50\sqrt{2} V$$



$$\bar{Z}_{23} = \bar{Z}_2 \parallel \bar{Z}_3 = \frac{\bar{Z}_2 \bar{Z}_3}{\bar{Z}_2 + \bar{Z}_3} = \frac{10(-j10)}{10 - j10}$$

$$\bar{Z}_{23} = \frac{-j10}{1-j} = \frac{10e^{-j\pi/2}}{\sqrt{2}e^{-j\pi/4}} = 5\sqrt{2}e^{-j\pi/4} \Omega$$

$$(\bar{Z}_{23} = (5 - j5) \Omega)$$

$$\bar{I}_1 = \frac{\bar{U}}{\bar{Z}_1 + \bar{Z}_{23}} = \frac{50\sqrt{2}}{j10 + 5 - j5} = \frac{50\sqrt{2}}{5(1+j)}$$

$$\bar{I}_1 = \frac{10\sqrt{2}}{\sqrt{2}e^{j\pi/4}} = 10e^{-j\pi/4} A$$

$$(\bar{I}_1 = (5\sqrt{2} - j5\sqrt{2}) A)$$

$$\bar{U}_2 = \bar{U}_3 = \bar{Z}_{23} \bar{I}_1 = 5\sqrt{2}e^{-j\pi/4} \cdot 10e^{-j\pi/4}$$

$$\bar{U}_2 = \bar{U}_3 = 50\sqrt{2}e^{-j\pi/2} V = -j50\sqrt{2} V$$

$$\bar{U}_1 = \bar{Z}_1 \bar{I}_1 = 10j\sqrt{2} \cdot 10e^{-j\pi/4}$$

$$\bar{U}_1 = 100e^{j\pi/4} V = (50\sqrt{2} + j50\sqrt{2}) V$$

$$\bar{I}_2 = \frac{\bar{U}_2}{\bar{Z}_2} = \frac{-j50\sqrt{2}}{10} = -j5\sqrt{2} A$$

$$\bar{I}_2 = 5\sqrt{2}e^{-j\pi/2} A$$

$$\bar{I}_3 = \frac{\bar{U}_3}{\bar{Z}_3} = \frac{-j50\sqrt{2}}{-j10} = 5\sqrt{2} A$$

$$\bar{I}_3 = 5\sqrt{2}e^{j0} A$$

b) $u_1(t) = 100\sqrt{2} \cos(\omega t + \pi/4) V$

$$c) \bar{S}_{23} = \bar{Z}_{23} I_1^2 = 5\sqrt{2} e^{-j\pi/4} \cdot 10^2 = 500\sqrt{2} e^{-j\pi/4} \text{ VA}$$

$$\bar{S}_{23} = (500 - j500) \text{ VA}$$

$$(i.e.) \bar{S}_{23} = \bar{U}_2 \bar{I}_1^* = 50\sqrt{2} e^{-j\pi/2} \cdot 10 e^{+j\pi/4} = 500\sqrt{2} e^{-j\pi/4} \text{ VA}$$

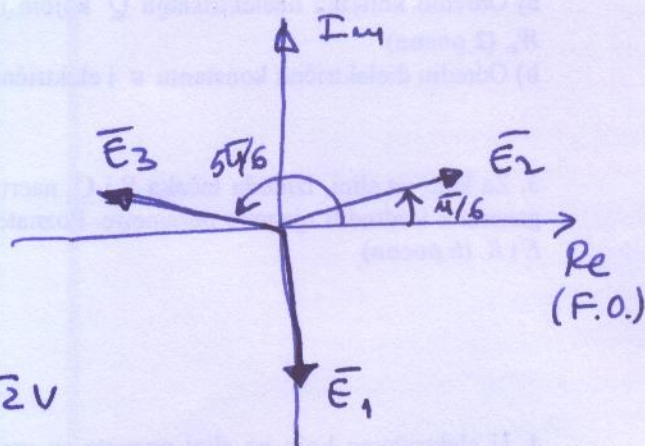
... inua joS machine ... $\bar{S}_{23} = \bar{S}_2 + \bar{S}_3 = \dots$

$$\rightarrow P = 500 \text{ W (AKTIVNA)}, \quad Q = -500 \text{ VAR (REAKTIVNA)}, \quad S = 500\sqrt{2} \text{ VA (PRIVIDNA)}$$

$$⑤ a) \bar{E}_1 = \frac{400}{\sqrt{2}} e^{j\pi/2} \text{ V} = 200\sqrt{2} e^{j\pi/2} \text{ V}$$

$$\bar{E}_2 = 200\sqrt{2} e^{j\pi/6} \text{ V}$$

$$\bar{E}_3 = 200\sqrt{2} e^{j5\pi/6} \text{ V}$$



$$b) U_L = |\bar{E}_1| = |\bar{E}_2| = |\bar{E}_3| = 200\sqrt{2} \text{ V}$$

$$Y: U_F = \frac{U_L}{\sqrt{3}} = \frac{200\sqrt{2}}{\sqrt{3}} \text{ V} = 200\sqrt{2/3} \text{ V}$$

$$Z = |\bar{Z}| = \sqrt{20^2 + 10^2} = 10\sqrt{5} \Omega$$

$$I_L = I_F = \frac{U_F}{Z} = \frac{200\sqrt{2}/\sqrt{3}}{10\sqrt{5}} = 20\sqrt{2/15} \text{ A}$$

$$\cos \varphi = \frac{P}{S} = \frac{R}{Z} = \frac{20}{10\sqrt{5}} = \frac{2}{\sqrt{5}}$$

$$Q = 3 X \cdot I_F^2 = 3 \cdot 10 \cdot 400 \cdot \frac{2}{15} = 1600 \text{ VAR} = 1,6 \text{ KVAR}$$