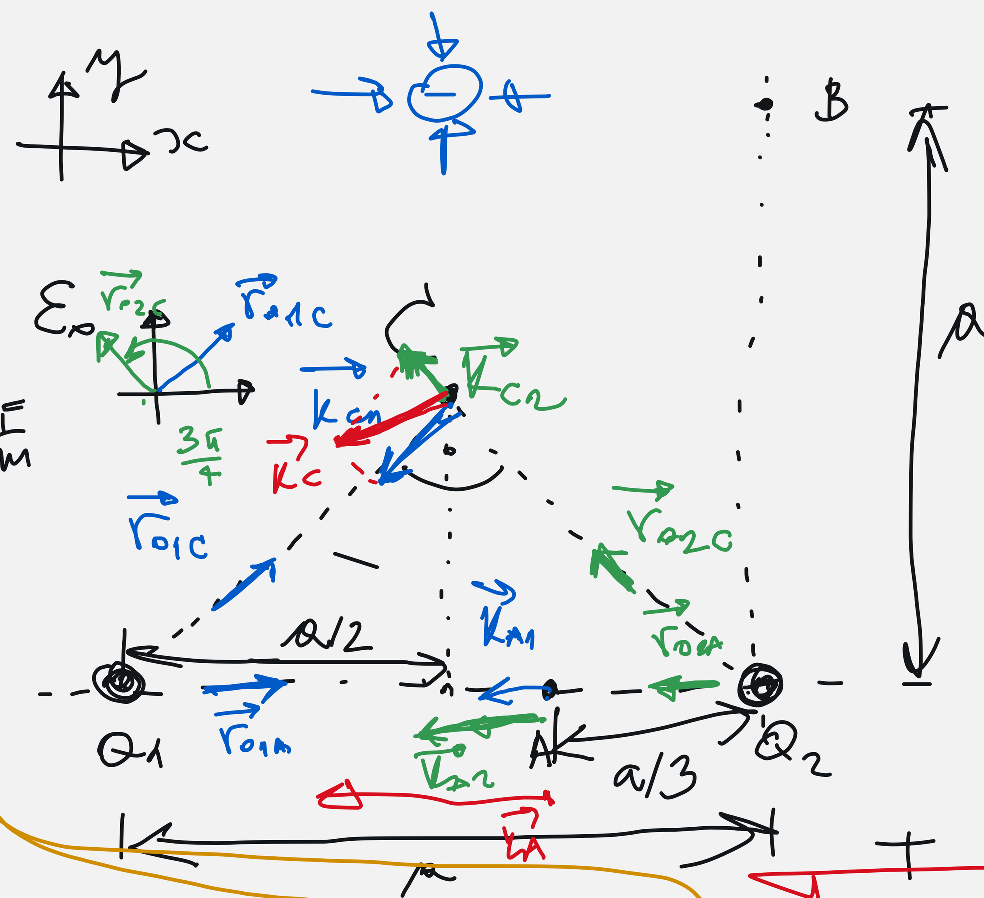


$\varepsilon_0 = 8{,}85\cdot 10^{-12}\,\frac{F}{m}$

$\varepsilon = \varepsilon_r \varepsilon_0$

$\varepsilon_{r_{vazd.}} \approx 1$

$Q_1 = -Q = -90\,\mu C$

$Q_2 = +\frac{Q}{2} = 45\,\mu C$

$a = 1\,mm$

a) $\vec{K}_A, \vec{K}_B, \vec{K}_C$

б) V_A, V_B, V_C, $U_{AB}, U_{AC} \ldots$

a) $\vec{K}_A = \vec{K}_{A1} + \vec{K}_{A2}$

$$\vec{K}_{A1} = \frac{Q_1}{4\pi\varepsilon_0\left(\frac{2a}{3}\right)^2}\,\underbrace{\vec{r}_{01A}}_{\vec{i}} = \frac{-9Q}{16\pi\varepsilon_0 a^2}\,\vec{i}$$

$$\vec{K}_{A2} = \frac{Q_2}{4\pi\varepsilon_0\left(\frac{a}{3}\right)^2}\,\underbrace{\vec{r}_{02A}}_{-\vec{i}} = \frac{-9Q}{8\pi\varepsilon_0 a^2}\,\vec{i}$$

$$\boxed{\vec{K}_A = -\frac{27Q}{16\pi\varepsilon_0 a^2}\,\vec{i}} = \ldots = -\_\,\frac{V}{m}\,\vec{i}$$

$$\vec{K}_C = \vec{K}_{C1} + \vec{K}_{C2}$$

$$\vec{K}_{C1} = \frac{Q_1}{4\pi\varepsilon_0\left(\frac{a}{\sqrt{2}}\right)^2}\,\vec{r}_{01C} = \frac{-Q}{2\pi\varepsilon_0 a^2}\Big(\underbrace{\cos\tfrac{\pi}{4}}_{\sqrt{2}/2}\,\vec{i} + \underbrace{\sin\tfrac{\pi}{4}}_{\sqrt{2}/2}\,\vec{j}\Big) = \frac{-Q\sqrt{2}}{4\pi\varepsilon_0 a^2}(\vec{i}+\vec{j})$$

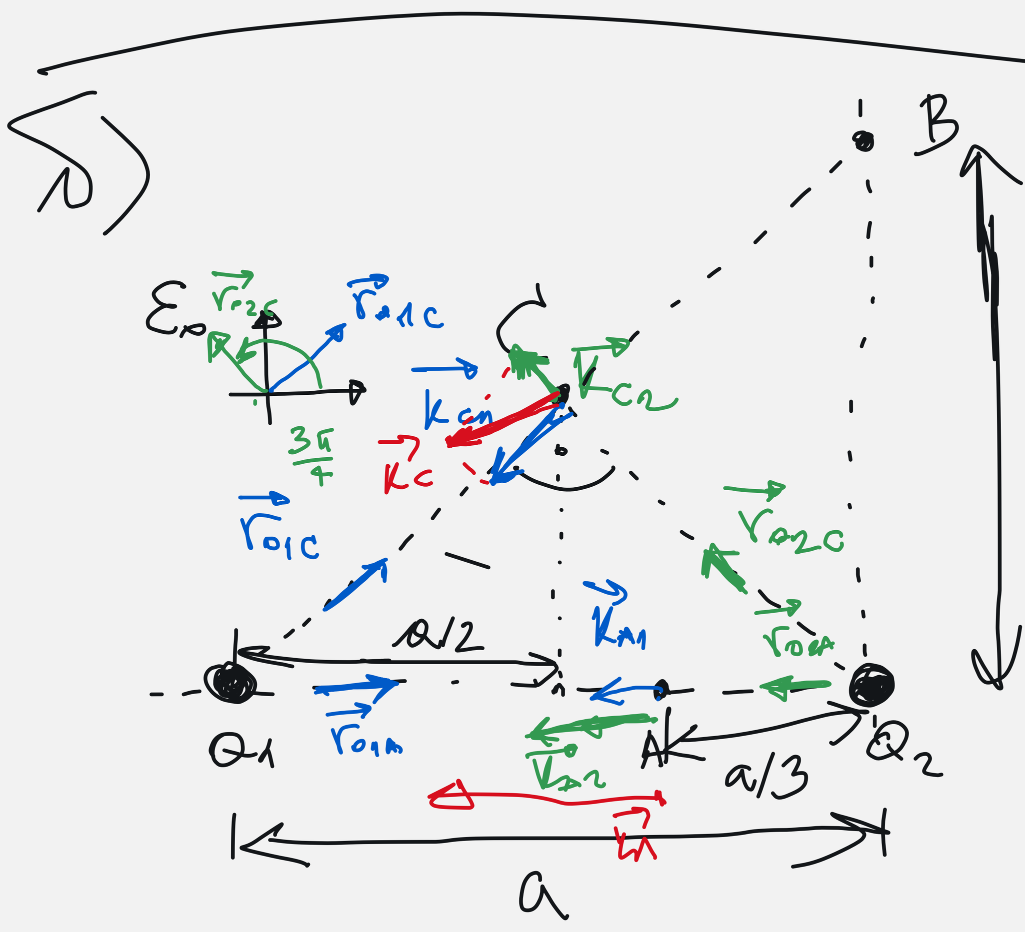
$$\vec{K}_{C2} = \frac{Q_2}{4\pi\varepsilon_0 (a/\sqrt{2})^2}\,\vec{r}_{02C} = \frac{+Q}{4\pi\varepsilon_0 a^2}\Big(\underbrace{\cos\tfrac{3\pi}{4}}_{-\sqrt{2}/2}\,\vec{i} + \underbrace{\sin\tfrac{3\pi}{4}}_{+\sqrt{2}/2}\,\vec{j}\Big) = \frac{+\sqrt{2}Q}{8\pi\varepsilon_0 a^2}(-\vec{i}+\vec{j})$$

$$\vec{K}_C = \frac{\sqrt{2}Q}{8\pi\varepsilon_0 a^2}\left(-2\vec{i}-2\vec{j}-\vec{i}+\vec{j}\right) = \frac{\sqrt{2}Q}{8\pi\varepsilon_0 a^2}\left(-3\vec{i}-\vec{j}\right)$$

$$\left(|\vec{K}_C| = \frac{\sqrt{2}Q}{8\pi\varepsilon_0 a^2}\cdot\sqrt{10} = \frac{Q\sqrt{5}}{4\pi\varepsilon_0 a^2}\right)$$

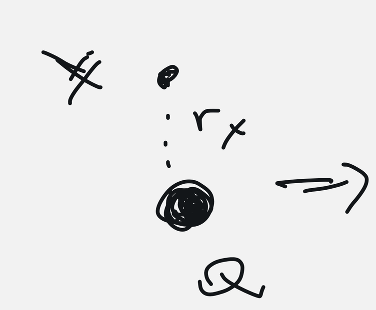
$\vec{K}_B = ?$ — "ДОМ." → конс. ...

б)



$V_A, V_B, V_C = ?$

$$V_x = \int_x^{\text{Ref.}} \vec{K}\cdot d\vec{l}$$



ref → ∞

$$V_x = \ldots = \frac{Q}{4\pi\varepsilon r_x}$$

$$V_A = V_{A1} + V_{A2}$$

$$V_{A1} = \frac{Q_1}{4\pi\varepsilon_0 \frac{2a}{3}} = \frac{-Q\cdot 3}{8\pi\varepsilon_0 a} = -\frac{3Q}{8\pi\varepsilon_0 a}$$

$$V_{A2} = \frac{Q_2}{4\pi\varepsilon_0 \frac{a}{3}} = \frac{Q/2}{\frac{4}{3}\pi\varepsilon_0 a} = +\frac{3Q}{8\pi\varepsilon_0 a}$$

$+ \quad V_A = \ldots = 0$

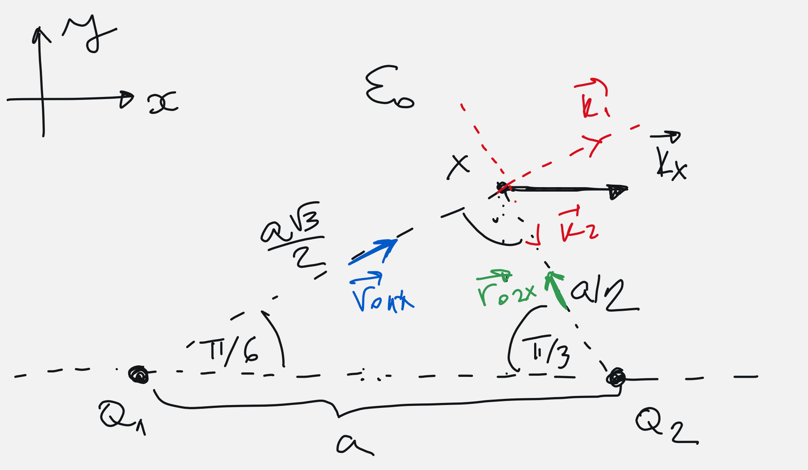
$$V_B = V_{B1} + V_{B2} = \frac{Q_1}{4\pi\varepsilon_0 a\sqrt{2}} + \frac{Q_2}{4\pi\varepsilon_0 a} = \frac{-Q}{4\sqrt{2}\pi\varepsilon_0 a} + \frac{Q}{8\pi\varepsilon_0 a} = \frac{Q}{4\pi\varepsilon_0 a}\left(-\frac{1}{\sqrt{2}}+\frac{1}{2}\right)$$

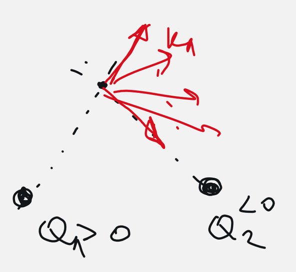
$$V_B = \frac{(-\sqrt{2}+1)Q}{8\pi\varepsilon_0 a} < 0$$

$$U_{AB} = V_A - V_B = 0 - \boxed{\ldots} = \frac{(\sqrt{2}-1)Q}{8\pi\varepsilon_0 a}$$

$V_C, U_{AC}, \ldots U_{CB}, \ldots$ "ДОМ"

$$\boxed{U_{BA} = -U_{AB}} \quad \left(V_B - \cancel{V_A}^{\,0} = V_B\right)$$





δ) $\frac{Q_1}{Q_2} = ?$ a) $Q_1 \gtrless 0$? $Q_2 \gtrless 0$?

$\boxed{\vec{K}_x = K_x^x \vec{i} + (0 \cdot \vec{j})}$

a) $Q_1 > 0$, $Q_2 < 0$ ✓

δ) $\vec{K}_x = \vec{K}_{x1} + \vec{K}_{x2}$

$\vec{K}_{x1} = \frac{Q_1}{4\pi\varepsilon_0 (\frac{a\sqrt{3}}{2})^2} \vec{r}_{01x} = \frac{Q_1}{3\pi\varepsilon_0 a^2} (\underbrace{\cos\frac{\pi}{6}}_{\sqrt{3}/2} \vec{i} + \underbrace{\sin\frac{\pi}{6}}_{1/2} \vec{j})$

$\vec{K}_{x2} = \frac{Q_2}{4\pi\varepsilon_0 (\frac{a}{2})^2} \vec{r}_{02x} = \frac{Q_2}{\pi\varepsilon_0 a^2} (\underbrace{\cos\frac{2\pi}{3}}_{-1/2} \vec{i} + \underbrace{\sin\frac{2\pi}{3}}_{\sqrt{3}/2} \vec{j})$

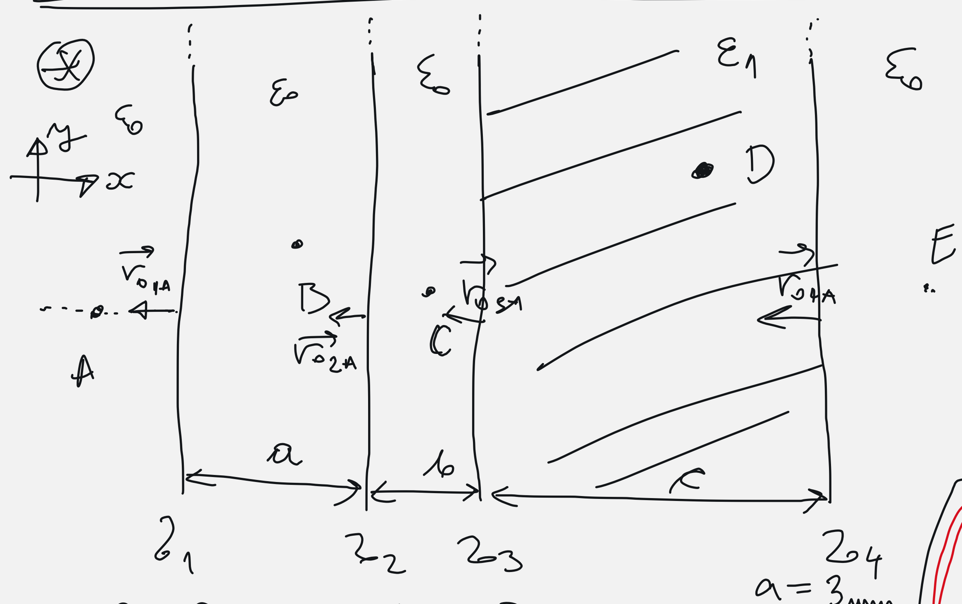
$\vec{K}_x = \vec{K}_{x1} + \vec{K}_{x2} = \dots = K_x^x \cdot \vec{i} + \underbrace{K_x^y}_{0} \cdot \vec{j}$

$K_x^y = K_{x1}^y + K_{x2}^y =$

$= \frac{1}{2} \cdot \frac{Q_1}{3\pi\varepsilon_0 a^2} + \frac{\sqrt{3}}{2} \frac{Q_2}{\pi\varepsilon_0 a^2} = 0$

$\frac{1}{\cancel{2}} \frac{Q_1}{3\cancel{\pi\varepsilon_0 a^2}} = -\frac{\sqrt{3}}{\cancel{2}} \frac{Q_2}{\cancel{\pi\varepsilon_0 a^2}}$

$\boxed{\frac{Q_1}{Q_2} = -3\sqrt{3}}$



$\varepsilon_0 = 8{,}85 \cdot 10^{-12} \frac{F}{m}$

$\varepsilon_1 = 10^{-10} \frac{F}{m}$

a) $\vec{K}_A, \vec{K}_B, \vec{K}_C, \vec{K}_D, \vec{K}_E = ?$

δ) $U_{12}, U_{34}, U_{42} = ?$

$\eta_1 = \eta = 10\,nC/m^2$ $\quad \eta_3 = 2\eta$

$\eta_2 = -\eta/2$ $\quad \eta_4 = -3\eta$

a = 3 mm
b = 2 mm
c = 5 mm

$\vec{K}_{A3} = \frac{\eta_3}{2\varepsilon_0} \vec{r}_{03A} = \frac{2\eta}{2\varepsilon_0} (-\vec{i})$

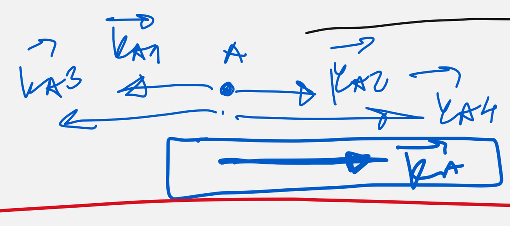
$\vec{K}_{A4} = \frac{\eta_4}{2\varepsilon_0} \vec{r}_{04A} = \frac{-3\eta}{2\varepsilon_0} (-\vec{i})$

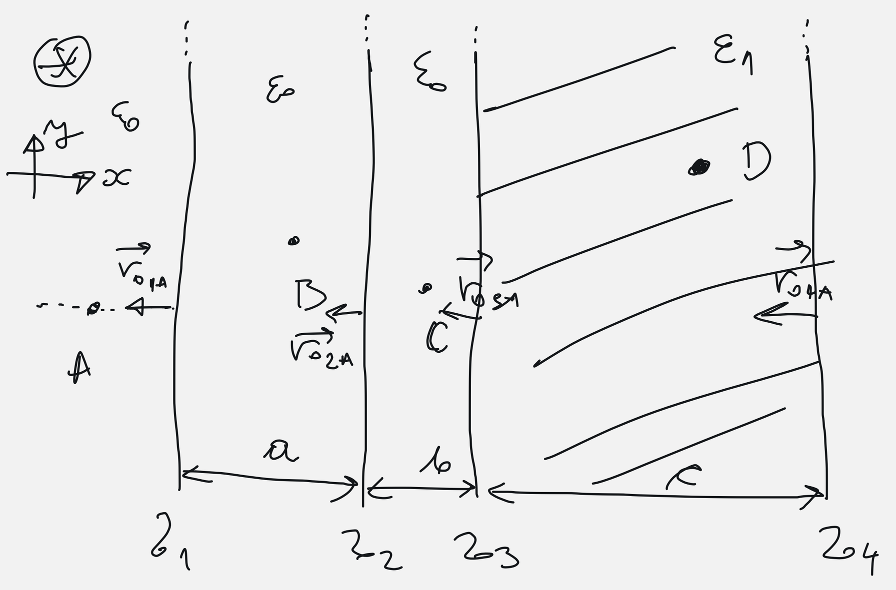
a) $\vec{K}_A = \vec{K}_{A1} + \vec{K}_{A2} + \vec{K}_{A3} + \vec{K}_{A4}$

$\vec{K}_{A1} = \frac{\eta_1}{2\varepsilon_0} \vec{r}_{01A} = \frac{\eta}{2\varepsilon_0}(-\vec{i})$; $\vec{K}_{A2} = \frac{\eta_2}{2\varepsilon_0} \vec{r}_{02A} = \frac{-\eta/2}{2\varepsilon_0}(-\vec{i})$

$\vec{K}_A = \dots = \frac{\eta_1 + \eta_2 + \eta_3 + \eta_4}{2\varepsilon_0} (-\vec{i})$

$\vec{K}_A = \frac{-\eta/2}{2\varepsilon_0} (-\vec{i}) = + \frac{\eta}{4\varepsilon_0} \vec{i}$





$\vec{K}_B = \vec{K}_{B1} + \vec{K}_{B2} + \vec{K}_{B3} + \vec{K}_{B4}$

$\vec{K}_{B1} = \frac{\eta_1}{2\varepsilon_0} \vec{r}_{01B} = \frac{\eta_1}{2\varepsilon_0} \vec{i}$

$\vec{K}_{B2} = \frac{\eta_2}{2\varepsilon_0} \vec{r}_{02B} = \frac{\eta_2}{2\varepsilon_0} (-\vec{i})$

$\vec{K}_{B3} = \frac{\eta_3}{2\varepsilon_0} (-\vec{i})$

$\vec{K}_{B4} = \frac{\eta_4}{2\varepsilon_0} (-\vec{i})$

$\vec{K}_B = \frac{1}{2\varepsilon_0} (\eta_1 - \eta_2 - \eta_3 - \eta_4) \vec{i}$

$\vec{K}_B = \frac{1}{2\varepsilon_0} (\eta - (-\frac{\eta}{2}) - 2\eta - (-3\eta)) \vec{i}$

$\vec{K}_B = \frac{2{,}5\eta}{2\varepsilon_0} \vec{i}$

$\vec{K}_C = \vec{K}_{C1} + \vec{K}_{C2} + \vec{K}_{C3} + \vec{K}_{C4} = \frac{\eta_1}{2\varepsilon_0} \vec{i} + \frac{\eta_2}{2\varepsilon_0} \vec{i} + \frac{\eta_3}{2\varepsilon_0}(-\vec{i}) + \frac{\eta_4}{2\varepsilon_0}(-\vec{i}) =$

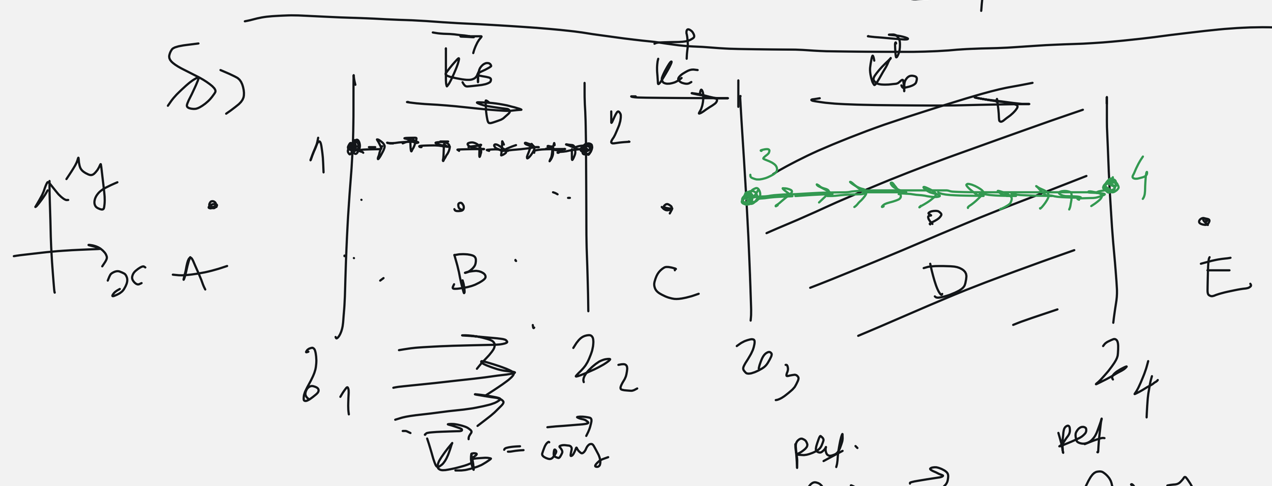
$= \frac{1}{2\varepsilon_0} (\eta_1 + \eta_2 - \eta_3 - \eta_4) = \frac{1{,}5\eta}{2\varepsilon_0} \vec{i}$

$\vec{K}_D = \vec{K}_{D1} + \vec{K}_{D2} + \vec{K}_{D3} + \vec{K}_{D4} = \frac{\eta_1}{2\varepsilon_1} \vec{i} + \frac{\eta_2}{2\varepsilon_1} \vec{i} + \frac{\eta_3}{2\varepsilon_1} \vec{i} + \frac{\eta_4}{2\varepsilon_1} (-\vec{i}) =$

$= \frac{\eta_1 + \eta_2 + \eta_3 - \eta_4}{2\varepsilon_1} \vec{i} = \frac{5{,}5\eta}{2\varepsilon_1} \vec{i}$

$\vec{K}_E = -\vec{K}_A$

δ)



$U_{12}, U_{34}, U_{42} = ?$

$U_{12} = V_1 - V_2 = \int_1^{ref} \vec{K} \cdot \vec{dl} - \int_2^{ref} \vec{K} \cdot \vec{dl} = \int_1^R + \int_R^2 = \int_1^2 \vec{K} \cdot \vec{dl}$

$\underline{\vec{i} \cdot \vec{i} = 1}$

$U_{12} = \int_1^2 \vec{K}_B \cdot \vec{dl} = \vec{K}_B \cdot \int_1^2 \vec{dl} = \vec{K}_B \cdot \vec{12} = \frac{2{,}5\eta}{2\varepsilon_0} \cdot \vec{i} \cdot a\vec{i}$

$U_{12} = \frac{2{,}5\eta}{2\varepsilon_0} \cdot a \; (= K_B \cdot a) = \dots = \_\_ V$

$U_{21} = \vec{K}_B \cdot \vec{21} = K_B \cdot a(-\vec{i}) = -K_B \cdot a = -U_{12}$!!

$U_{34} = \dots = \vec{K}_D \cdot \vec{34} = K_D \vec{i} \cdot c \vec{i} = K_D \cdot c = \frac{5{,}5\eta}{2\varepsilon_1} \cdot c = \dots = \_\_ V$

$U_{42} = -U_{24} = -(U_{23} + U_{34}) = -(K_C \cdot b + K_D \cdot c) = \dots$

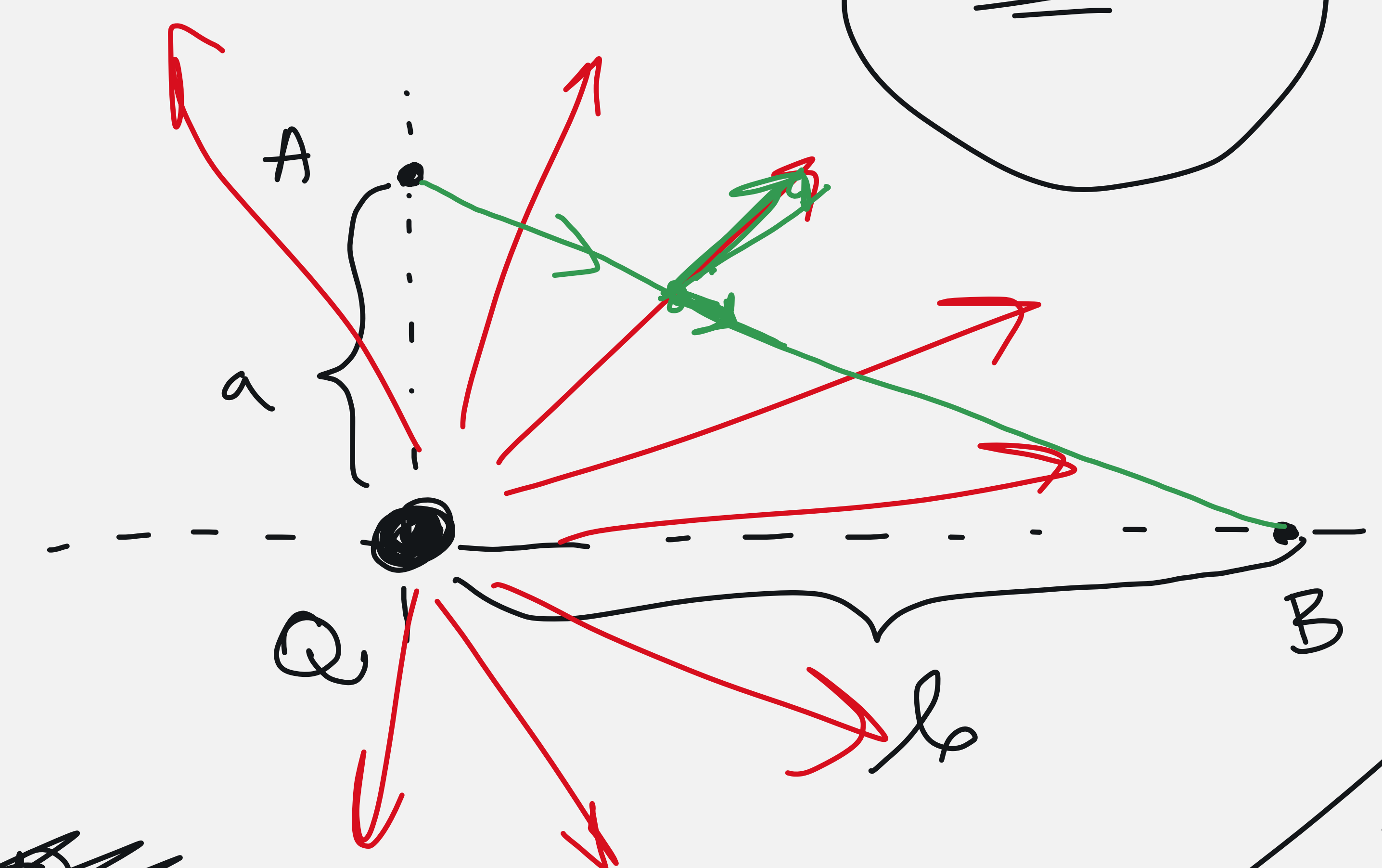
$U_{24} = \int_2^4 \vec{K} \cdot \vec{dl} = \int_2^3 + \int_3^4 = U_{23} + U_{34}$

$+V_2 - V_4$

$U_{42} = \int_4^2 \vec{K} \cdot \vec{dl} = \int_4^3 + \int_3^2 = \vec{K}_D \cdot \vec{43} + \vec{K}_C \cdot \vec{32} =$

$= K_D \vec{i} \cdot c(-\vec{i}) + K_C \vec{i} \cdot b(-\vec{i})$

$= -K_D \cdot c - K_C \cdot b$



$\boxed{\varepsilon_r}$

$U_{AB} = ?$

$Q,\ a,\ b,\ \varepsilon_r$

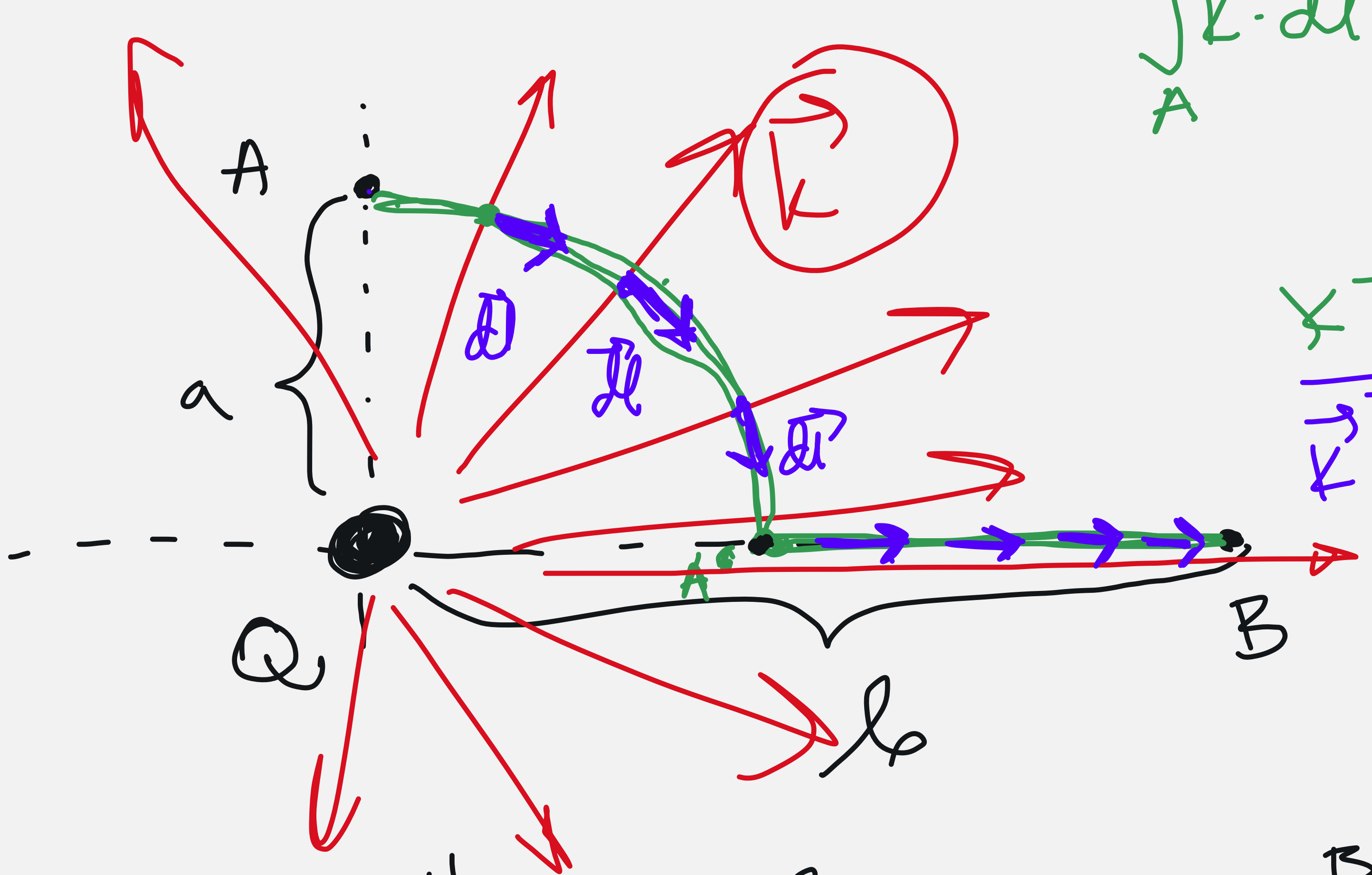
1° $U_{AB} = V_A - V_B = \frac{Q}{4\pi\varepsilon_0 a} - \frac{Q}{4\pi\varepsilon_0 b}$

$$U_{AB} = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{a} - \frac{1}{b}\right) = \frac{(b-a)\,Q}{4\pi\varepsilon_0\, a\cdot b}$$

2° $U_{AB} = \int_A^B \vec{K}\cdot d\vec{l}$

$$\vec{K}\cdot d\vec{l} = |\vec{K}|\cdot|d\vec{l}|\cdot\cos\angle\,\vec{K}, d\vec{l}$$

Hajgrace ϕ u $\frac{\pi}{2}$



$$\int_A^B \vec{K}\cdot d\vec{l} = \int_A^{A'} + \int_{A'}^B$$

$\angle\,\vec{K}, d\vec{l} = \frac{\pi}{2}$

$\vec{K}\cdot d\vec{l} = 0$

$\angle\,\vec{K}, d\vec{l} = 0$

$\vec{K}\cdot d\vec{l} = K\cdot dl\cdot \cancel{\cos 0}\ 1$

$= K\cdot dl$

$$U_{AB} = \int_A^B \vec{K}\cdot d\vec{l} = \int_A^{A'} \vec{K}\cdot d\vec{l} + \int_{A'}^B \vec{K}\cdot d\vec{l} = 0 + \int_{A'}^B K\cdot dl =$$

$$= \int_{A'}^B \frac{Q}{4\pi\varepsilon r^2}\cdot dr = \frac{Q}{4\pi\varepsilon}\int_{r=a}^{r=b}\frac{dr}{r^2} = \frac{Q}{4\pi\varepsilon}\left(-\frac{1}{r}\right)\Big|_a^b = \frac{Q}{4\pi\varepsilon}\left(\frac{1}{a} - \frac{1}{b}\right)$$