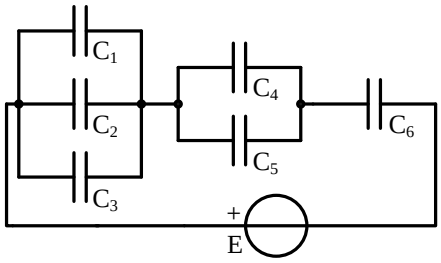
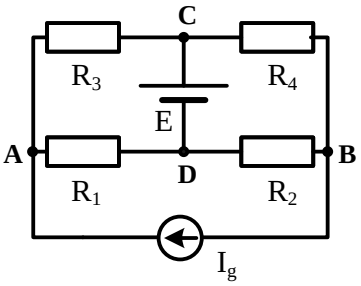
Elektrotehnika - pismeni ispit

13. mart 2026.

1. U kolu na Slici 1 poznate su kapacitivnosti $C_1 = C = 20\,\mu\text{F}$, $C_2 = 2C$, $C_3 = C_4 = C_5 = C_6 = 3C$, i elektromotorna sila idealnog naponskog generatora $E = 2\,\text{V}$. Pre povezivanja, svi kondenzatori su bili neopterećeni. Odrediti naelektrisanje na C_3, napon na C_5 i ukupnu energiju svih kondenzatora. (**8 poena**)



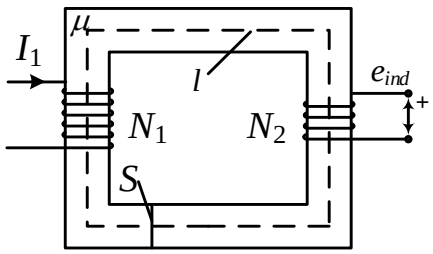
Slika 1



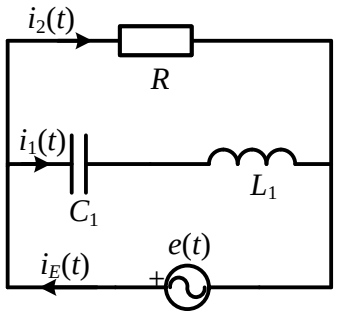
Slika 2

2. U kolu na Slici 2 poznati su parametri: $R_1 = R_3 = 2\,\Omega$, $R_2 = R_4 = 4\,\Omega$, $E = 40\,\text{V}$, $I_g = 10\,\text{A}$. Primenom metode potencijala čvorova ili metode konturnih struja odrediti intenzitete struja kroz generator E i otpornik R_2 (samostalno zadati referentni smer), a zatim odrediti snagu ovih elemenata. **(12 poena)**

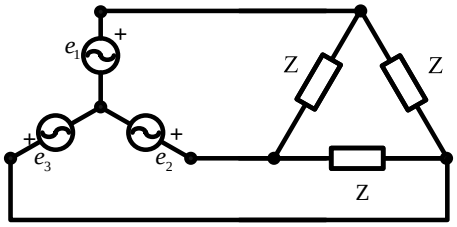
3. U kolu na Slici 3 prikazano je magnetno kolo sa dva namotaja. Kroz namotaj sa $N_1 = 25$ navojaka protiče konstantna struja $I_1 = 12\,\text{A}$, a krajevi namotaja sa $N_2 = 50$ navojaka su otvoreni. Jezgro je površine poprečnog preseka $S = 5\,\text{cm}^2$, dužine srednje linije $l = 20\,\text{cm}$ i magnetne permeabilnosti $\mu = 2 \cdot 10^{-4}\,\text{H/m}$.
a) Odrediti fluks vektora magnetne indukcije i jačinu magnetnog polja u jezgru. **(3 poena)**
b) Odrediti energiju magnetnog polja u jezgru. **(3 poena)**
c) Odrediti međusobnu iduktivnost namotaja. **(2 poena)**
d) Odrediti indukovanu elektromotornu silu na krajevima otvorenog namotaja. **(2 poena)**



Slika 3



Slika 4

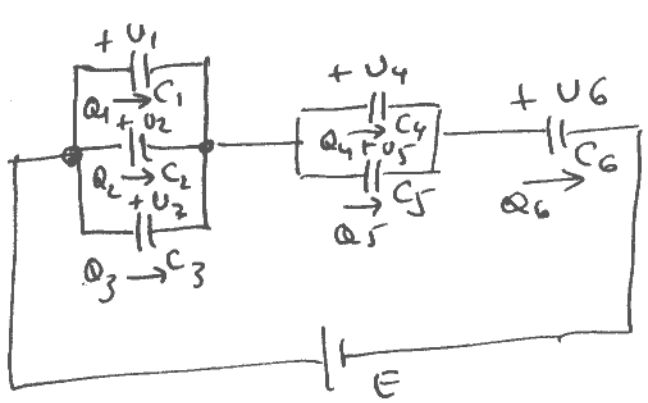


Slika 5

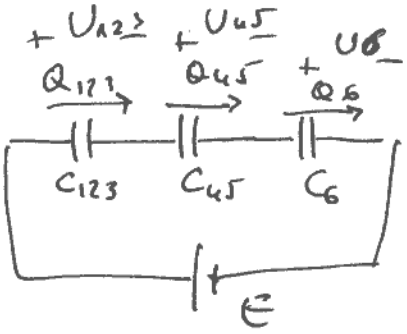
4. U kolu na Slici 4 poznato je: $e(t) = 200\sqrt{2}\cos(500t - \pi)\,\text{V}$, $C_1 = 80\,\mu\text{F}$, $L_1 = 40\,\text{mH}$, $R = 5\Omega$. Odrediti:
a) kompleksne struje u granama i nacrtati fazorski dijagram svih struja, **(5 poena)**
b) vremenski oblik napona na induktivnom kalemu, **(2 poena)**
c) aktivnu, reaktivnu i prividnu snagu i faktor snage celokupnog potrošača, **(4 poena)**
d) kompleksnu admitansu potrošača. **(1 poen)**

5. Na Slici 5 prikazan je trofazni sistem generator-potrošač. Efektivna vrednost elektromotornih sila generatora iznosi $E = 150\,\text{V}$, aktivna snaga potrošača $P = 2.7\,\text{kW}$, a reaktivna snaga Q = -3.6 kVAr. Odrediti: efektivnu vrednost linijske struje i kompleksnu impedansu potrošača. **(8 poena)**

(1)



$\Leftrightarrow$

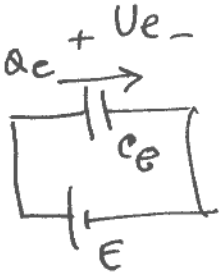


$\Leftrightarrow$

$C_{123} = C_1 + C_2 + C_3 = 6\,C = 120\,\mu C$

$C_{45} = C_4 + C_5 = 6C = 120\,\mu C$

$\Leftrightarrow$



$$C_e = \frac{1}{\frac{1}{C_{123}} + \frac{1}{C_{45}} + \frac{1}{C_6}} = \frac{1}{\frac{1}{6C} + \frac{1}{6C} + \frac{1}{3C}} = \frac{1}{\frac{4}{6C}} = \frac{3}{2}C$$

$C_e = 30\,\mu C$

$U_e - \mathcal{E} = 2V$

$Q_e = C_e \cdot U_e = \frac{3}{2}C\mathcal{E} = \frac{3}{2} \cdot 20\mu \cdot 2 = 60\,\mu C = Q_{123} = Q_{45} = Q_6$

$$U_{45} = \frac{Q_{45}}{C_{45}} = \frac{\frac{3}{2}C\mathcal{E}}{6C} = \frac{1}{4}\mathcal{E} = 0{,}5V = U_4 = U_5$$

$$Q_4 = C_4 U_{45} = 3C \cdot \frac{\mathcal{E}}{4} = \frac{3}{4}C\mathcal{E} = 30\,\mu C$$

$$U_{123} = \frac{Q_{123}}{C_{123}} = \frac{\frac{3}{2}C\mathcal{E}}{6C} = \frac{1}{4}\mathcal{E} = 0{,}5V = U_1 = U_2 = U_3$$

$$Q_3 = C_3 U_3 = 3C \cdot \frac{1}{4}\mathcal{E} = \frac{3}{4}C\mathcal{E} = 30\,\mu C$$

$$W_{Ce} = \frac{1}{2} C_e U_e^2 = \frac{1}{2}\,\frac{3}{2}C \cdot \mathcal{E}^2 = \frac{3}{4}C\mathcal{E}^2 = 60\,\mu J$$

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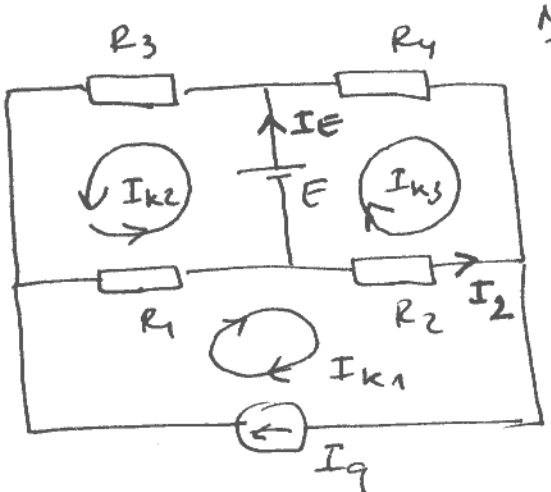
(2)



MKS

(1) $I_{K1} = I_g = 10\,A$

(2) $R_{21} I_{K1} + R_{22} I_{K2} + R_{23} I_{K3} = \sum_{K2} \mathcal{E}_i$

(3) $R_{31} I_{K1} + R_{32} I_{K2} + R_{33} I_{K3} = \sum_{K3} \mathcal{E}_i$

(2) $R_1 I_{K1} + (R_1 + R_3) I_{K2} + 0\, I_{K3} = \mathcal{E}$

(3) $-R_2 I_{K1} + 0 \cdot I_{K2} + (R_2 + R_4) I_{K3} = \mathcal{E}$

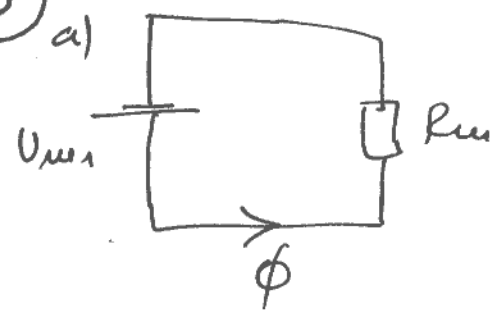
(2) $2 \cdot 10 + 4\, I_{K2} = 40 \Rightarrow I_{K2} = 5\,A$

(3) $-1 \cdot 10 + 8\, I_{K3} = 40 \Rightarrow I_{K3} = 10\,A$

$I_E = I_{K2} + I_{K3} = 15\,A \Rightarrow P_E = \mathcal{E} \cdot I_E = 40 \cdot 15 = 600\,W$

$I_2 = I_{K1} - I_{K3} = 0\,A \Rightarrow P_{R2} = R_2 I_2^2 = 0\,W$

(3) a)



$$U_{m1} = N_1 I_1 \qquad \phi = \frac{U_{m1}}{R_m}$$

$$R_m = \frac{l}{\mu S} \qquad \phi = \frac{N_1 I_1}{\frac{l}{\mu S}}$$

$$\phi = \frac{\mu S N_1 I_1}{l} = \frac{2\cdot 10^{-4}\cdot 5\cdot 10^{-4}\cdot 25\cdot 12}{20\cdot 10^{-2}}$$

$$\phi = 150\cdot 10^{-6}\,Wb = 150\,\mu Wb$$

$$B = \frac{\phi}{S} = \frac{\mu N_1 I_1}{l} = 0{,}3\,T$$

$$H = \frac{B}{\mu} = \frac{N_1 I_1}{l} = 1500\,A/m$$

b) $w = \frac{1}{2} BH$

$$W = w\cdot V = \frac{1}{2} BH\cdot l\cdot S = \frac{1}{2}\cdot 0{,}3\cdot 1500\cdot 20\cdot 10^{-2}\cdot 5\cdot 10^{-4} = 22{,}5\,mJ$$

c)

$$|L_{12}| = |L_{21}| = \frac{\Psi_{12}}{I_1} = \frac{N_2\phi}{I_1} = \frac{N_1 N_2}{R_m} = 625\,\mu H$$

d)

$$e_{ind} = -\frac{d\Psi}{dt} = 0\,V, \quad \text{fluks} = \text{const!}$$

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(4) a)

$$\underline{E} = 200\, e^{\pm j\pi} = -200\,V$$

$$\underline{Z}_R = R = 5\,\Omega$$

$$\underline{Z}_{C1} = -j\frac{1}{\omega C_1} = -j\frac{1}{500\cdot 80\cdot 10^{-6}} = -j25\,\Omega = 25\, e^{-j\pi/2}\,\Omega$$

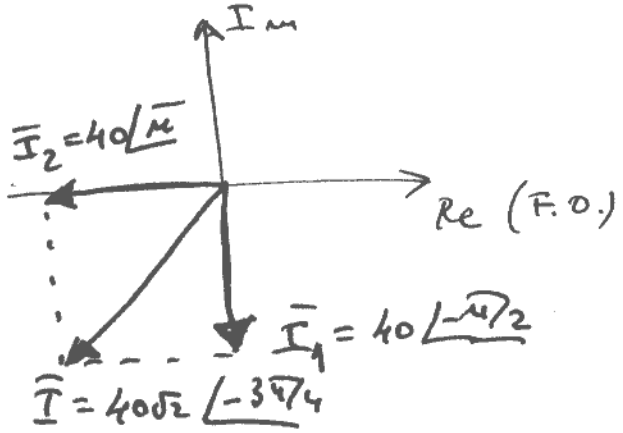
$$\underline{Z}_{L1} = j\omega L_1 = j\,500\cdot 40\cdot 10^{-3} = j20\,\Omega = 20\, e^{j\pi/2}\,\Omega$$

$$\underline{Z}_1 = \underline{Z}_{L1} + \underline{Z}_{C1} = j20 - j25 = -j5\,\Omega = 5\, e^{-j\pi/2}\,\Omega$$

$$\underline{I}_1 = \frac{\underline{E}}{\underline{Z}_1} = \frac{200\, e^{-j\pi}}{5\, e^{-j\pi/2}} = 40\, e^{-j\pi/2}\,A = -j40\,A$$

$$\underline{I}_2 = \frac{\underline{E}}{\underline{Z}_R} = \frac{-200}{5} = -40\,A = 40\, e^{j\pi}\,A$$

$$\bar{I}_E = \bar{I}_1 + \bar{I}_2 = (-40 - j40)\,A = 40\sqrt{2}\, e^{-j\,3\pi/4}\,A$$



b) $\bar{U}_{L1} = \bar{Z}_{L1} \cdot \bar{I}_1 = 20 e^{j\pi/2} \cdot 40 e^{-j\pi/2} = 800 e^{j0}\,V$

$u_{L1}(t) = 800\sqrt{2}\cos(500t)\,V$

c) $\bar{S}_{POT} = \bar{S}_{gen} = \bar{E} \cdot \bar{I}_E^* = 200 e^{-j\pi} \cdot 40\sqrt{2}\, e^{j3\pi/4} = 8000\sqrt{2}\, e^{-j\pi/4}\,VA$

$\bar{S}_{POT} = 8000\sqrt{2}\left(\underbrace{\cos(-\pi/4)}_{1/\sqrt{2}} + j\underbrace{\sin(-\pi/4)}_{-1/\sqrt{2}}\right) = (8000 - j8000)\,VA$

$P_{POT} = 8000\,W$, $Q_{POT} = -8000\,VAR$, $S_{POT} = 8000\sqrt{2}\,VA$

$\cos\varphi = 1/\sqrt{2}$

d) $\bar{Y} = \dfrac{\bar{I}_E}{\bar{E}} = \dfrac{-40 - j40}{-200} = (0{,}2 + j0{,}2)\,S$

$\left(\bar{Y} = \dfrac{1}{\bar{Z}_1} + \dfrac{1}{\bar{Z}_2} = \dfrac{1}{-j5} + \dfrac{1}{5} = (0{,}2 + j0{,}2)\,S\right)$

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(5) $U_L = \sqrt{3}\,E = 150\sqrt{3}\,V$ (generatori, Y)

$U_F = U_L = 150\sqrt{3}\,V$ (potrošači, Δ)

$S = 3U_F \cdot I_F \Rightarrow I_F = \dfrac{S}{3U_F} = \dfrac{4500}{3 \cdot 150\sqrt{3}} = 10/\sqrt{3}\,A$

$S = \sqrt{P^2 + Q^2}$

$S = \sqrt{2700^2 + 3600^2}$

$S = \sqrt{900^2 \cdot (3^2 + 4^2)}$

$S = 900 \cdot 5 = 4500\,VA$

(potrošači, Δ) $I_L = \sqrt{3}\,I_F = 10\,A$

$Z = \dfrac{U_F}{I_F} = \dfrac{150\sqrt{3}}{10/\sqrt{3}} = 45\,\Omega$

$\cos\varphi = \dfrac{P}{S} = 3/5$

$\sin\varphi = \dfrac{Q}{S} = -4/5$

$\bar{Z} = Z\cos\varphi + jZ\sin\varphi = (27 - j36)\,\Omega$

$$\bar{S} = P + jQ = 3\,\bar{Z}\cdot I_f^2$$

$$\bar{Z} = \frac{P+jQ}{3I_f^2} = \frac{2700 - j3600}{3\cdot\left(\frac{10}{\sqrt{3}}\right)^2} = (27 - j36)\,\Omega$$