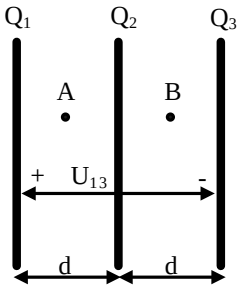
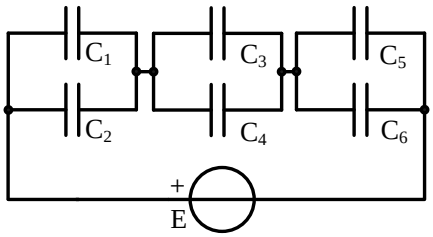
Elektrotehnika - pismeni ispit

27. april 2026.

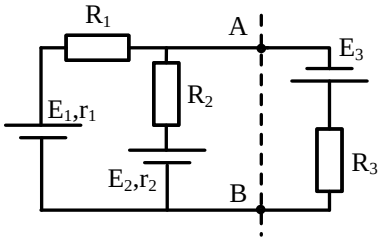
1. Tri veoma velike ravnomerno naelektrisane metalne ploče, jednakih površina $S = 100\ \mathrm{m}^2$, nalaze se u sredini čija je dielektrična konstanta $\varepsilon = 2 \cdot 10^{-9}\ \mathrm{F/m}$, na međusobnom rastojanju od $d = 2\ \mathrm{cm}$, kao na Slici 1. Ploče su opterećene količinama naelektrisanja: $Q_1 = 2\ \mu\mathrm{C}$, $Q_2 = 8\ \mu\mathrm{C}$ i $Q_3 = -4\ \mu\mathrm{C}$.
a) Usvojiti koordinatni sistem, a zatim odrediti i nacrtati vektor električnog polja u tačkama A i B. (**6 p.**)
b) Odrediti napon između prve i treće ploče U_{13}. (**2 poena**)



Slika 1



Slika 2

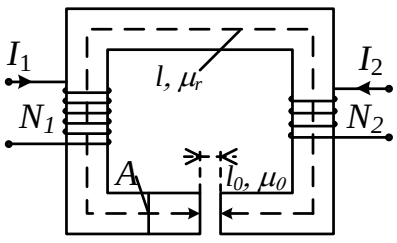


Slika 3

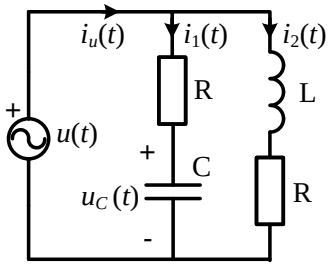
2. U kolu na Slici 2 poznate su kapacitivnosti $C_1 = 10\ \mu\mathrm{F}$, $C_2 = 20\ \mu\mathrm{F}$, $C_3 = C_4 = 30\ \mu\mathrm{F}$, $C_5 = 50\ \mu\mathrm{F}$, $C_6 = 10\ \mu\mathrm{F}$ i elektromotorna sila naponskog generatora $E = 40\ \mathrm{V}$. Pre povezivanja, svi kondenzatori su bili neopterećeni. Odrediti naelektrisanje na C_1, napon na C_3 i energiju kondenzatora C_5. (**8 poena**)

3. U kolu na Slici 3 poznati su parametri: $R_1 = 30\ \Omega$, $R_2 = 50\ \Omega$, $R_3 = 20\ \Omega$, $E_1 = 50\ \mathrm{V}$, $r_1 = 10\ \Omega$, $E_2 = 10\ \mathrm{V}$, $r_2 = 10\ \Omega$ i $E_3 = 10\ \mathrm{V}$.
a) Nacrtati Tevenenov generator za deo kola levo od tačaka A i B i odrediti njegove parametre. **(8 poena)**
b) Primenom Tevenenove teoreme odrediti intenzitet struje kroz otpornik R_3 i napon U_{AB}. **(2 poena)**

4. Na Slici 4 prikazano je magnetno kolo sa dva namotaja. Kroz namotaj sa N_1 navojaka protiče struja I_1, a kroz namotaj sa N_2 navojaka struja I_2. Jezgro se sastoji od feromagnetnog materijala površine poprečnog preseka A, dužine srednje linije l i relativne magnetne permeabilnosti μ_r, i vazdušnog procepa debljine l_0, kao što je ilustrovano na Slici 4. Odrediti:
a) intenzitet vektora magnetne indukcije i jačine magnetnog polja u vazdušnom procepu, **(4 poena)**
b) fluks vektora magnetne indukcije u feromagnetnom materijalu jezgra, **(2 poena)**
c) energiju magnetnog polja u feromagnetnom materijalu jezgra, **(2 poena)**
d) sopstvene induktivnosti i međusobnu iduktivnost namotaja. **(4 poena)**



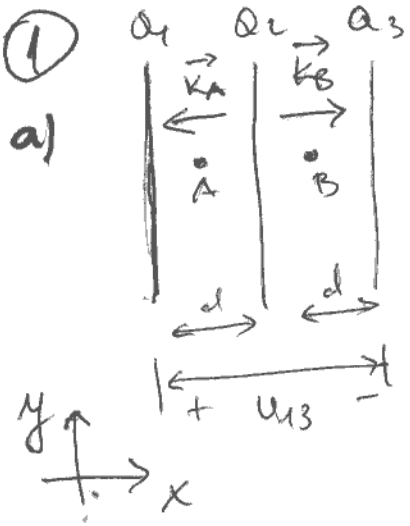
Slika 4



Slika 5

5. Na Slici 5 je prikazano kolo naizmenične struje poznatih parametara: $R = 25\ \Omega$, $L = 25\ \mathrm{mH}$, $C = 40\ \mu\mathrm{F}$ i $u(t) = 100\sqrt{2}\cos(1000t + \pi/2)\ \mathrm{V}$. Odrediti:
a) kompleksne izraze za sve označene struje u kolu i skicirati njihov fazorski dijagram, **(6 poena)**
b) vremenski oblik napona na kondenzatoru, **(2 poena)**
c) kompleksnu impedansu celokupnog potrošača, **(1 poen)**
d) aktivnu, reaktivnu i prividnu snagu celokupnog potrošača. **(3 poena)**

①



a)

$$\vec{K_A} = \vec{K_{A1}} + \vec{K_{A2}} + \vec{K_{A3}} = \frac{\sigma_1}{2\varepsilon}\vec{i} + \frac{\sigma_2}{2\varepsilon}(-\vec{i}) + \frac{\sigma_3}{2\varepsilon}(-\vec{i})$$

$$\vec{K_A} = \frac{Q_1 - Q_2 - Q_3}{2\varepsilon S}\vec{i} = \frac{(2-8+4)\cdot 10^{-6}}{2\cdot 2\cdot 10^{-9}\cdot 100}\vec{i}$$

$$\vec{K_A} = -\frac{1}{2}\cdot 10^{1}\vec{i} = -5\vec{i}\ V/m$$

$$\vec{K_B} = \vec{K_{B1}} + \vec{K_{B2}} + \vec{K_{B3}} = \frac{\sigma_1}{2\varepsilon}\vec{i} + \frac{\sigma_2}{2\varepsilon}\vec{i} + \frac{\sigma_3}{2\varepsilon}(-\vec{i})$$

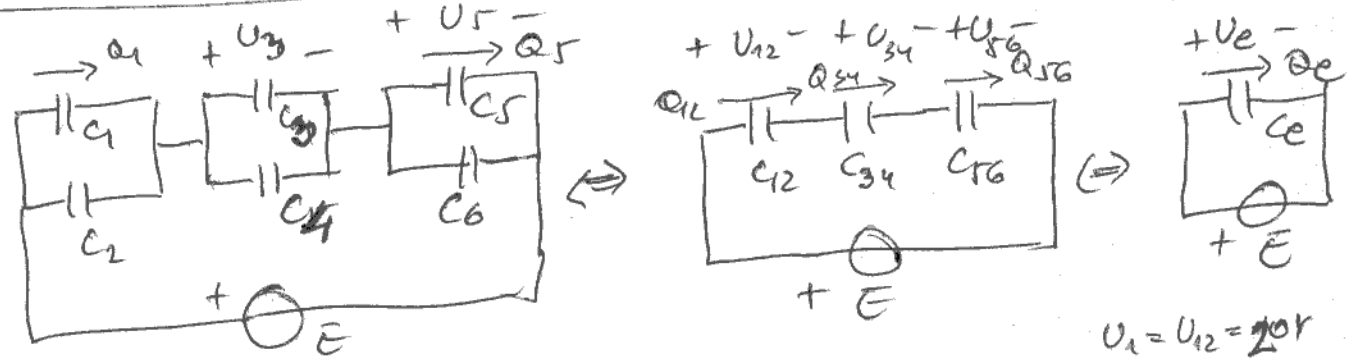
$$\vec{K_B} = \frac{Q_1 + Q_2 - Q_3}{2\varepsilon S}\vec{i} = \frac{(2+8+4)\cdot 10^{-6}}{2\cdot 2\cdot 10^{-9}\cdot 100}\vec{i}$$

$$\vec{K_B} = 35\vec{i}\ V/m$$

b) $U_{13} = U_{12} + U_{23} = \vec{K_A}\cdot(d\cdot\vec{i}) + \vec{K_B}\cdot(d\cdot\vec{i}) = -5\cdot 0{,}02 + 35\cdot 0{,}02 = 0{,}6V$

---

②



$C_{12} = C_1 + C_2 = 30\,\mu F$

$C_{34} = C_3 + C_4 = 60\,\mu F$

$C_{56} = C_5 + C_6 = 60\,\mu F$

$$C_e = \frac{1}{\frac{1}{C_{12}} + \frac{1}{C_{34}} + \frac{1}{C_{56}}}$$

$$C_e = \frac{1}{\frac{1}{30} + \frac{1}{60} + \frac{1}{60}} = 15\,\mu F$$

$U_e = E = 40V$

$Q_e = C_e U_e = 600\,\mu C$

$Q_{12} = Q_{34} = Q_{56} = Q_e = 600\,\mu C$

$$U_{12} = \frac{Q_{12}}{C_{12}} = \frac{600\mu}{30\mu} = 20V$$

$$U_{34} = \frac{Q_{34}}{C_{34}} = 10V$$

$$U_{56} = \frac{Q_{56}}{C_{56}} = 10V$$

$U_1 = U_{12} = 20V$

$Q_1 = C_1 U_1 = 200\,\mu C$

$U_3 = U_{34} = 10V$

$U_5 = U_{56} = 10V$

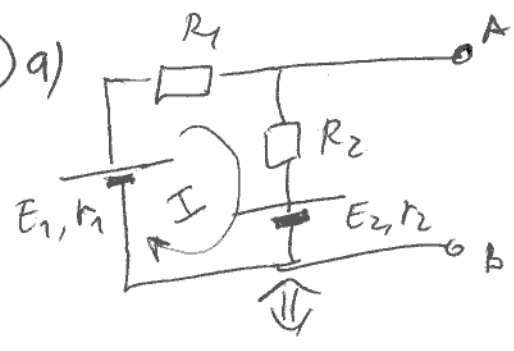
$W_5 = \frac{1}{2} C_5 U_5^2$

$W_5 = 2500\,\mu J$

$W_5 = 2{,}5\,mJ$

---

③ a)

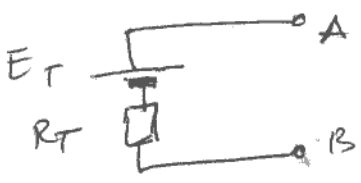


$E_T = U_{AB}^{OV}$:

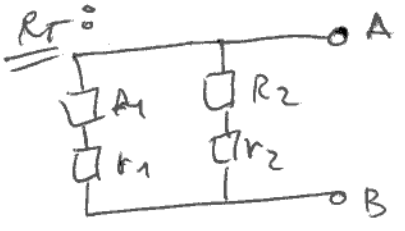
$$-E_1 + R_1 I + r_1 I + R_2 I + E_2 + r_2 I = 0$$

$$I = \frac{E_1 - E_2}{R_1 + R_2 + r_1 + r_2} = \frac{50 - 10}{30 + 50 + 10 + 10} = 0{,}4A$$

$$E_T = U_{AB}^{OV} = R_2 I + r_2 I + E_2 = 34V$$



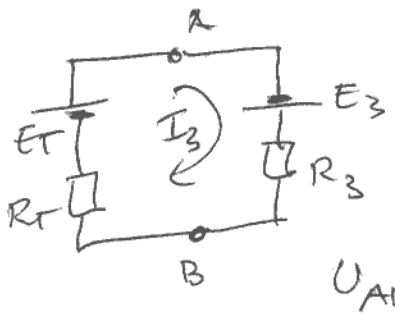
R_T:



$$R_T = R_{AB}^{OV} = \frac{(R_1 + r_1)(R_2 + r_2)}{R_1 + R_2 + r_1 + r_2}$$

$$R_T = \frac{40\cdot 60}{100} = 24\,\Omega$$

b)



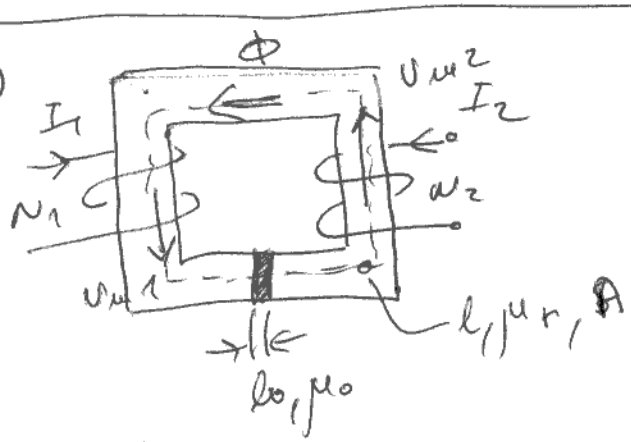
$$-E_T + R_T I_3 - E_3 + R_3 I_3 = 0$$

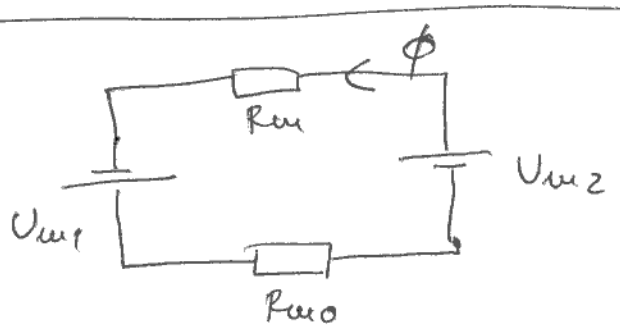
$$I_3 = \frac{E_T + E_3}{R_T + R_3} = \frac{34+10}{24+20} = 1A$$

$$U_{AB} = -E_3 + R_3 I_3 = -10 + 20 \cdot 1 = 10V$$

---

④





$$U_{m1} = N_1 I_1 \qquad R_m = \frac{l}{\mu_0 \mu_r A}$$

$$U_{m2} = N_2 I_2 \qquad R_{m0} = \frac{l_0}{\mu_0 A}$$

a, b)

$$\Phi = \frac{U_{m1} + U_{m2}}{R_m + R_{m0}}$$

$$\Phi = \frac{(N_1 I_1 + N_2 I_2)/\mu_0 A}{(l/\mu_r + l_0)}$$

$$\mathcal{B}_0 = B = \frac{\Phi}{A} = \frac{\mu_0 (N_1 I_1 + N_2 I_2)}{l_0 + l/\mu_r}$$

$$H_0 = \frac{B_0}{\mu_0} = \frac{N_1 I_1 + N_2 I_2}{l_0 + l/\mu_r}$$

c)

$$W = w \cdot V, \quad w = \frac{1}{2} BH, \quad V = l_0 A$$

$$W = \frac{1}{2} \frac{B^2}{\mu_0 \mu_r} l_0 A = \frac{\mu_0^2 (N_1 I_1 + N_2 I_2)^2}{2 \mu_0 \mu_r (l_0 + l/\mu_r)^2} \cdot lA$$

$$W = \frac{\mu_0 l A}{2 \mu_r} \left(\frac{N_1 I_1 + N_2 I_2}{l_0 + l/\mu_r} \right)^2$$

d)

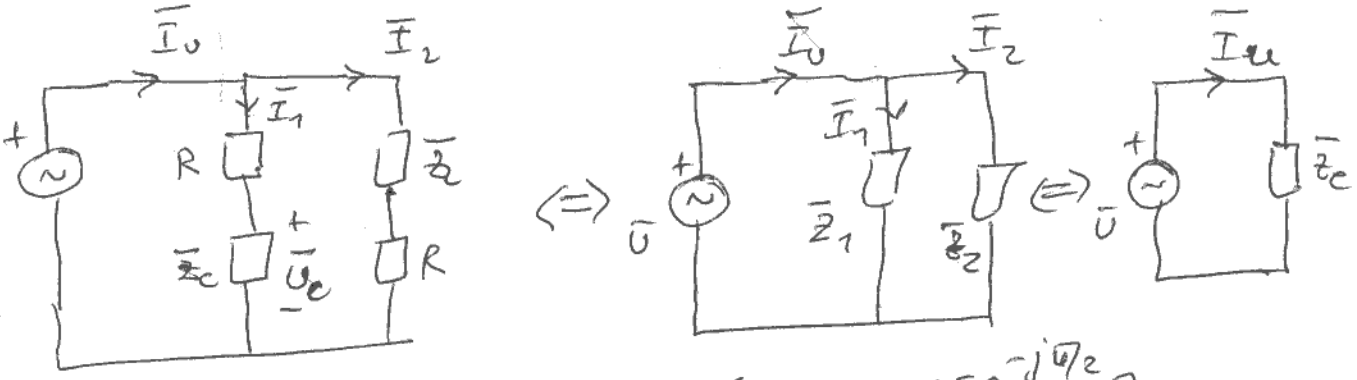
$$L_1 = \left. \frac{\Psi_1}{I_1} \right|_{I_2=0} = \frac{N_1 \Phi (I_2=0)}{I_1} = \frac{N_1^2}{R_m + R_{m0}} = \frac{\mu_0 A N_1^2}{l_0 + l/\mu_r}$$

$$L_2 = \left. \frac{\Psi_2}{I_2} \right|_{I_1=0} = \frac{N_2 \Phi (I_1=0)}{I_2} = \frac{N_2^2}{R_m + R_{m0}} = \frac{\mu_0 A N_2^2}{l_0 + l/\mu_r}$$

$$L_{12} = \left. \frac{\Psi_2}{I_1} \right|_{I_2=0} = \left. \frac{\Psi_1}{I_2} \right|_{I_1=0} = \frac{N_1 N_2}{R_m + R_{m0}} = \frac{\mu_0 A N_1 N_2}{l_0 + l/\mu_r}$$

5

a)



$$\bar{Z}_c = -j\frac{1}{\omega C} = -j\frac{1}{1000 \cdot 40 \cdot 10^{-6}} = -j25\,\Omega = 25e^{-j\pi/2}\,\Omega$$

$$\bar{Z}_L = j\omega L = j1000 \cdot 25 \cdot 10^{-3} = j25\,\Omega = 25e^{j\pi/2}\,\Omega$$

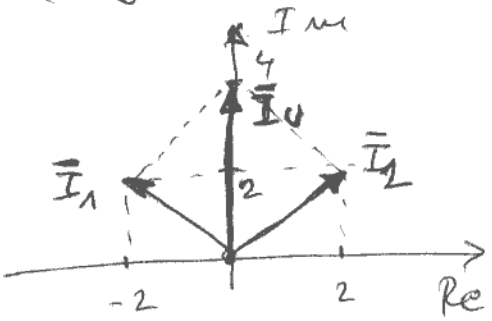
$$\bar{Z}_1 = R + \bar{Z}_c = (25 - j25)\,\Omega = 25\sqrt{2}e^{-j\pi/4}\,\Omega$$

$$\bar{Z}_2 = R + \bar{Z}_c = (25 + j25)\,\Omega = 25\sqrt{2}e^{j\pi/4}\,\Omega$$

$$\bar{I}_1 = \frac{\bar{U}}{\bar{Z}_1} = \frac{100e^{j\pi/2}}{25\sqrt{2}e^{j\pi/4}} = 2\sqrt{2}e^{j3\pi/4} = 2\sqrt{2}\left(-\frac{1}{\sqrt{2}} + j\frac{1}{\sqrt{2}}\right) = (-2 + j2)\,A$$

$$\bar{I}_2 = \frac{\bar{U}}{\bar{Z}_2} = \frac{100e^{j\pi/2}}{25\sqrt{2}e^{j\pi/4}} = 2\sqrt{2}e^{j\pi/4} = (2 + j2)\,A$$

$$\bar{I}_0 = \bar{I}_1 + \bar{I}_2 = j4\,A = 4e^{j\pi/2}\,A$$



b) $$\bar{U}_c = \bar{Z}_c \cdot \bar{I}_1 = 25e^{-j\pi/2} \cdot 2\sqrt{2}e^{j3\pi/4}$$

$$\bar{U}_c = 50\sqrt{2}\,e^{j\pi/4}\,V$$

$$u_c(t) = 100\cos(1000t + \pi/4)\,V$$

c) $$\bar{Z}_e = \frac{\bar{Z}_1 \cdot \bar{Z}_2}{\bar{Z}_1 + \bar{Z}_2} = \frac{(25 - j25)(25 + j25)}{25 - j25 + 25 + j25} = \frac{25^2(1-j)(1+j)}{25(1-j+1+j)} = \frac{25(1^2 - j^2)}{2}$$

$$\bar{Z}_e = 25\,\Omega$$

d) $$\bar{S} = \bar{U} \cdot \bar{I}_0^* = \bar{Z}_e I_0^2 = \frac{U^2}{\bar{Z}_e^*} = (400 + j0)\,VA$$

$$P = 400\,W$$

$$Q = 0\,VAR$$

$$S = |\bar{S}| = \sqrt{P^2 + Q^2} = 400\,VA$$