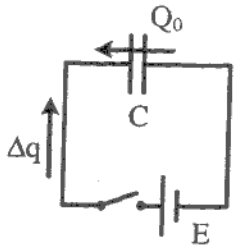
Elektrotehnika – teorijski deo ispita

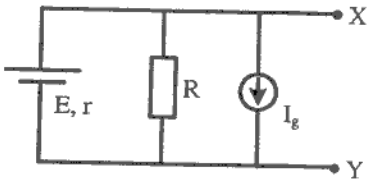
27. maj 2026.

1. (6 poena) Odrediti količinu naelektrisanja Δq koja će proteći nakon zatvaranja prekidača, ako je kondenzator kapacitivnosti $C = 5\,\text{nF}$ prethodno bio opterećen količinom naelektrisanja $Q_0 = 20\,\text{nC}$. Ems generatora iznosi $E = 20\,\text{V}$. Koliko iznosi rad generatora, A_E, nakon zatvaranja prekidača?

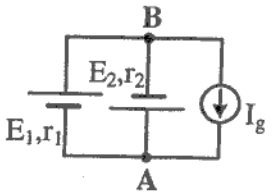


2. (6 poena) Pločasti kondenzator je priključen na idealni naponski generator elektromotorne sile $E = 2\,\text{V}$ i pri tome je poznato da energija elektrostatičkog polja kondenzatora iznosi $W_C = 2\,\mu\text{J}$. Ako je kondenzator sve vreme priključen na izvor napajanja, a ploče kondenzatora razmaknu, tako da je rastojanje između njih duplo veće od početnog, odrediti kapacitivnost, naelektrisanje i energiju kondenzatora nakon razmicanja ploča.

3. (6 poena) Za kolo na slici, između tačaka X i Y, nacrtati ekvivalentni **Nortonov generator** i odrediti njegove parametre, I_N i G_N.
Vrednosti parametara su: $E = 90\,\text{V}$, $r = 30\,\Omega$, $I_g = 1\,\text{A}$ i $R = 60\,\Omega$.

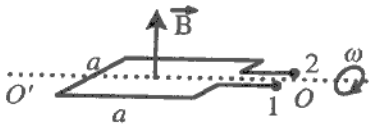


4. (7 poena) U električnom kolu na slici poznate su vrednosti parametara: $E_1 = 20\,\text{V}$, $E_2 = 30\,\text{V}$, $r_1 = r_2 = 10\,\Omega$ i $I_g = 5\,\text{A}$.
Primenom **teoreme superpozicije**, odrediti najpre napon U_{AB}, a zatim odrediti snagu strujnog generatora.

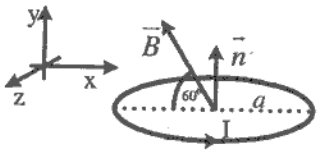


Studenti koji su položili 1. kolokvijum i rade popravni 2. kolokvijum, treba da rade samo zadatke 5, 6, 7 i 8

5. (6 poena) Kvadratna kontura stranice a nalazi se u homogenom magnetnom polju konstantnog vektora magnetne indukcije $\vec{B}$. Kontura rotira ugaonom brzinom ω oko ose OO' koja se nalazi u ravni konture i upravna je na vektor $\vec{B}$, kao što je prikazano na slici. Odrediti indukovanu elektromotornu silu između priključaka konture 1 i 2.



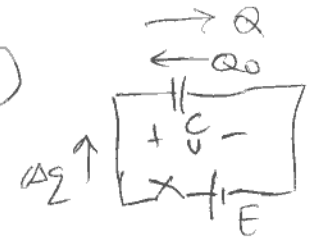
6. (6 poena) Na slici je prikazana kruta ravna kružna provodna kontura, poluprečnika a, koja se nalazi u horizontalnoj ravni paralelnoj sa X i Z osom. Kroz konturu protiče struja intenziteta I. Kontura je u homogenom magnetnom polju vektora magnetne indukcije $\vec{B}$ koji se nalazi u ravni paralelnoj sa X i Y osom, a sa ravni konture zaklapa ugao od 60°, kao na slici. Odrediti vektor momenta sile kojim polje deluje na konturu u skladu sa zadatim koordinatnim sistemom.



7. (6 poena) Trofazni potrošač, koji se sastoji od tri impedanse $\bar{Z} = 10e^{-j\pi/3}\,\Omega$ povezane u trougao, priključen je na simetričan trofazni sistem napona. Odrediti efektivnu vrednost linijske struje i reaktivnu snagu trofaznog potrošača, ako je poznato da je aktivna snaga 1.5 kW.

8. Kalemu induktivnosti $L = 5\,\text{mH}$ priključen je na naponski generator $u(t) = 100\sqrt{2}\cos(2000t + 3\pi/4)\,\text{A}$.
Odrediti:
a) (1 poen) kompleksnu impedansu i admitansu kalema,
b) (3 poena) kompleksni i vremenski oblik struje kalema (referentni smer struje je usaglašen sa naponom),
c) (3 poena) aktivnu, reaktivnu i prividnu snagu kalema.

(1)



$$\left.\begin{aligned} Q &= CU = CE \\ Q &= -Q_0 + \Delta Q \end{aligned}\right\} \quad CE = +\Delta Q - Q_0$$

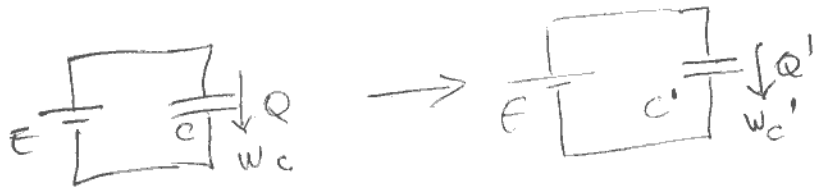
$$\Delta Q = Q_0 + CE = 20\mu + 1\mu \cdot 20$$

$$\boxed{\Delta Q = 120\,\mu C}$$

$$A_E = \Delta Q \cdot E = 120\mu \cdot 20 = 2400\,\mu J = 2{,}4\,mJ$$

---

(2)



$$W_C = \frac{1}{2} C E^2$$

$$C = \frac{2W_C}{E^2} = \frac{2 \cdot 2\mu}{(2)^2} = 1\mu F$$

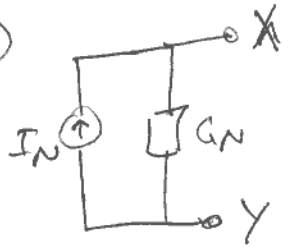
$$C' = \frac{\varepsilon S}{d'} = \frac{\varepsilon S}{2d} = \frac{C}{2} = 0{,}5\mu F$$

$$Q' = C'E = 0{,}5\mu \cdot 2 = 1\mu C$$

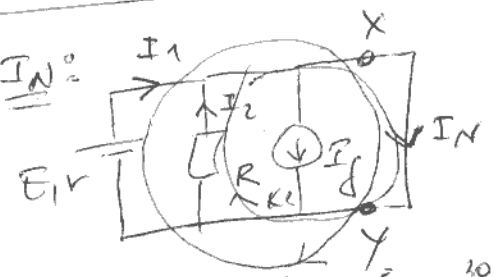
$$W_C' = \frac{1}{2} Q' E = \frac{1}{2} \cdot 1\mu \cdot 2 = 1\mu J$$

---

(3)



I_N:



$$I_N = I_1 + I_2 - I_g \quad (I_K \text{ z. } \angle AX)$$

K1: $E - rI_1 = 0 \Rightarrow I_1 = \frac{E}{r} = 3A$

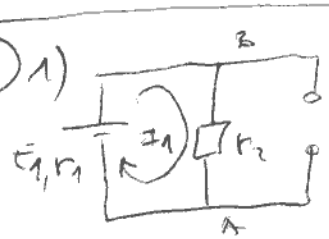
K2: $-RI_2 = 0 \Rightarrow I_2 = 0$

$$I_N = \frac{E}{r} - I_g = 3A - 1A = \mathbf{2A}$$

G_N: $R_N = r \| R \overset{R=1}{=} \frac{r \cdot R}{r+R} = \frac{30 \cdot 60}{90} = 20\Omega \Rightarrow G_N = \frac{1}{R_N} = 50\,mS$

---

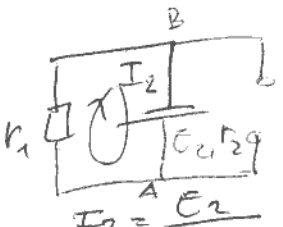
(4) 1)



$$I_1 = \frac{E_1}{r_1 + r_2} \qquad U_{AB} = -r_2 I_1$$

$$U_{AB}' = -\frac{r_2}{r_1 + r_2} E_1 = -10V$$

2)

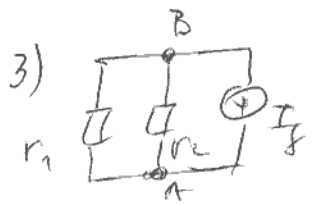


$$I_2 = \frac{E_2}{r_1 + r_2}$$

$$U_{AB} = \frac{r_1}{r_1 + r_2} E_2$$

$$U_{AB}'' = 15\,V$$

3)



$$U_{AB}''' = I_g \cdot (r_1 \| r_2)$$

$$U_{AB}''' = \frac{r_1 r_2}{r_1 + r_2} I_g = 25V$$

$$U_{AB} = U_{AB}' + U_{AB}'' + U_{AB}'''$$

$$U_{AB} = -10 + 15 + 25$$

$$U_{AB} = 30V$$

$$P_{I_g} = U_{AB} I_g$$

$$P_{I_g} = 30 \cdot 5 = 150W$$

---

(5)

$$\Phi = \vec{B} \cdot \vec{S} = BS \cos \sphericalangle(\vec{B}, \vec{S}) = Ba^2 \cos(\omega t)$$

$$e(t) = -\frac{d\Phi(t)}{dt} = -Ba^2 \frac{d}{dt}\left[\cos(\omega t)\right] = Ba^2 \omega \sin(\omega t)$$

---

(6)

$$\vec{\mu} = I \vec{S} \times \vec{B} = |I \vec{S} \times \vec{B}| \cdot \vec{k} = ISB \cdot \sin \sphericalangle(\vec{S}, \vec{B}) \cdot \vec{k}$$

$$\vec{\mu} = I a^2 \pi B \underbrace{\sin \sphericalangle(\vec{n}, \vec{B})}_{30^\circ} \cdot \vec{k} = \frac{I a^2 \pi B}{2} \cdot \vec{k}$$

---

(7)

$$\bar{Z} = 10 e^{-j\pi/3}\,\Omega \Rightarrow Z = 10, \quad \cos\varphi = \cos(-\pi/3) = \frac{1}{2}, \quad \sin(-\pi/3) = \sin\varphi = -\frac{\sqrt{3}}{2}$$

$$P = S\cos\varphi \Rightarrow S = \frac{P}{\cos\varphi} = \frac{1{,}5k}{1/2} = 3\,KVA = 3000\,VA$$

$$Q = S \sin\varphi \Rightarrow Q = 3k \cdot \left(-\frac{\sqrt{3}}{2}\right) = -1500\sqrt{3}\,VAr \qquad (\Delta \text{ potrošač})$$

$$S = 3 Z I_F^2 \Rightarrow I_F = \sqrt{S/3Z} = \sqrt{\frac{3000}{30}} = \sqrt{100} = 10A \Rightarrow I_L = \sqrt{3} I_F = 10\sqrt{3}\,A$$

(8) a) $\bar{Z}_L = j\omega L = j2000 \cdot 5\,m = j10\,\Omega = 10\, e^{j\pi/2}\,\Omega$

$\bar{Y}_L = \frac{1}{\bar{Z}_L} = \frac{1}{j10} = -j100m\,S = 100 e^{-j\pi/2}\,mS$

b.) $\bar{U} = 100 e^{j3\pi/4}\,V \quad \bar{I} = \frac{\bar{U}}{\bar{Z}_L} = \frac{100 e^{j3\pi/4}}{10 \cdot e^{j\pi/2}} = 10\, e^{j\pi/4}\,A$

$i(t) = 10\sqrt{2}\cos(2000t + \pi/4)\,A$

c) $\bar{S} = \bar{U} \cdot \bar{I}^* = 100 e^{j3\pi/4} \cdot 10 e^{-j\pi/4} = 1000 e^{j\pi/2} = j1000\,VA$

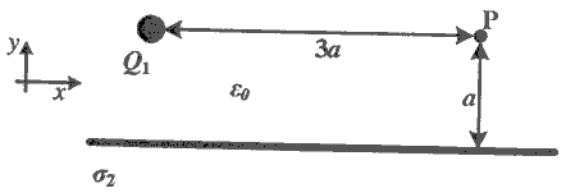
$P = 0\,W \quad Q = 1\,KVAR, \quad S = 1\,KVA$

Elektrotehnika - pismeni ispit

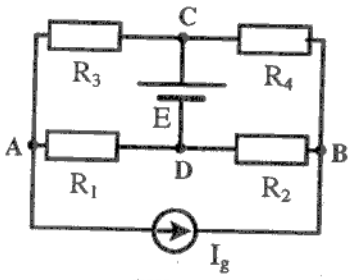
27. maj 2026.

1. Tačkasto naelektrisanje $Q_1 = Q > 0$ i veoma velika, ravnomerno naelektrisana površ, površinske gustine naleketrisanja $\sigma_2 = -\sigma < 0$ nalaze se u vazduhu, kao na Slici 1.

a) Odrediti i skicirati **vektor** elektičnog polja u tački P. **(4 poena)**
b) Odrediti i skicirati **vektor** sile kojom površ deluje na tačkasto naelektisanje. **(4 poena)**



Slika 1

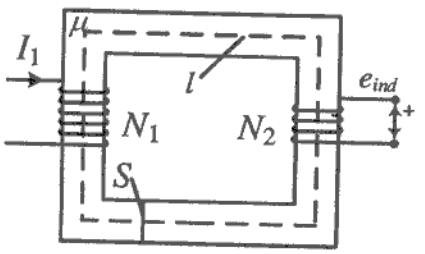


Slika 2

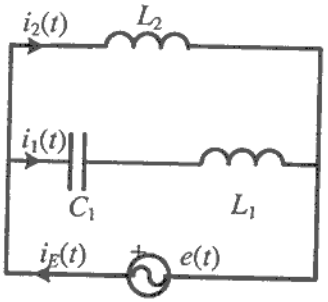
2. U kolu na Slici 2 poznati su parametri: $R_1 = R_3 = 2\,\Omega$, $R_2 = R_4 = 4\,\Omega$, $E = 20\,\text{V}$, $I_g = 10\,\text{A}$. Rešiti kolo primenom metode potencijala čvorova ili metode konturnih struja, a nakon toga odrediti snagu otpornika R_4 i snagu strujnog generatora I_g. **(12 poena)**

3. U kolu na Slici 3 prikazano je magnetno kolo sa dva namotaja. Kroz namotaj sa $N_1 = 50$ navojaka protiče konstantna struja $I_1 = 12\,\text{A}$, a krajevi namotaja sa $N_2 = 40$ navojaka su otvoreni. Jezgro je površine poprečnog preseka $S = 10\,\text{cm}^2$, dužine srednje linije $l = 50\,\text{cm}$ i magnetne permeabilnosti $\mu = 2\cdot 10^{-4}\,\text{H/m}$.

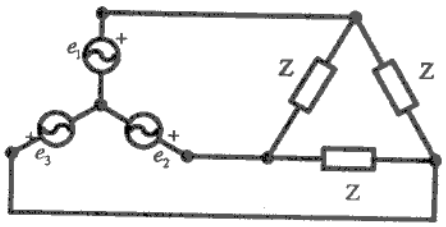
a) Odrediti fluks vektora magnetne indukcije i jačinu magnetnog polja u jezgru. **(4 poena)**
b) Odrediti energiju magnetnog polja u jezgru. **(3 poena)**
c) Odrediti indukovanu elektromotornu silu na krajevima otvorenog namotaja. **(3 poena)**



Slika 3



Slika 4



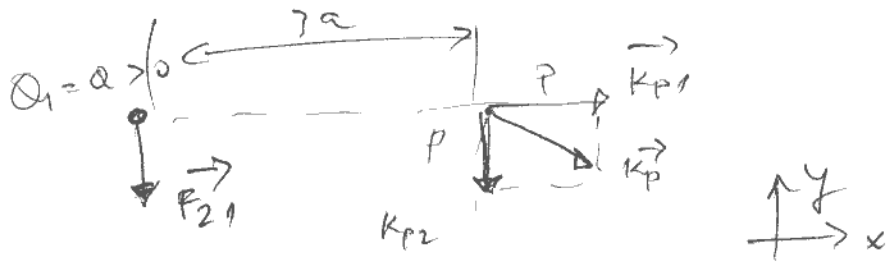
Slika 5

4. U kolu na Slici 4 poznato je: $e(t) = 100\sqrt{2}\cos(500t - \pi)\,\text{V}$, $C_1 = 40\,\mu\text{F}$, $L_1 = 50\,\text{mH}$, $L_2 = 40\,\text{mH}$. Odrediti:

a) kompleksne struje u svim granama i nacrtati njihov fazorski dijagram, **(4 poena)**
b) vremenski oblik napona na kondenzatoru, **(2 poena)**
c) aktivnu, reaktivnu i prividnu snagu i faktor snage celokupnog potrošača, **(4 poena)**
d) kompleksnu admitansu potrošača. **(2 poena)**

5. Na Slici 5 prikazan je trofazni sistem generator-potrošač. Efektivna vrednost elektromotornih sila generatora iznosi $E = 500\,\text{V}$, fazna struja potrošača (pretežno kapacitivnog) $I_f = \sqrt{3}\,\text{A}$, a aktivna snaga $P = 2.7\,\text{kW}$. Odrediti faktor snage i kompleksnu impedansu potrošača. **(8 poena)**

① a)



$\sigma_2 = -\sigma < 0$

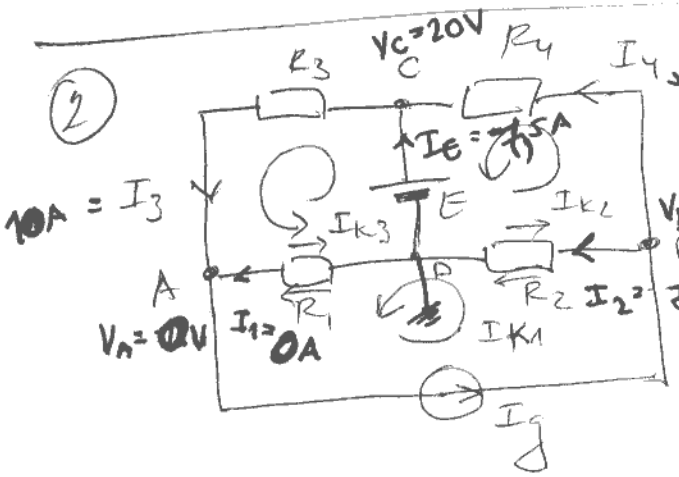
$$\vec{K}_P = \vec{K}_{P1} + \vec{K}_{P2} = \frac{Q_1}{4\pi\varepsilon_0 (3a)^2}\vec{i} + \frac{\sigma_2}{2\varepsilon_0}\vec{j}$$

$$\vec{K}_P = \frac{Q}{36\pi\varepsilon_0 a^2}\vec{i} - \frac{\sigma}{2\varepsilon_0}\vec{j}$$

b) $$\vec{F}_{21} = \vec{K}_2 \cdot Q_1 = \left(\frac{\sigma_2}{2\varepsilon_0}\vec{j}\right)\cdot Q_1 = -\frac{Q\sigma}{2\varepsilon_0}\vec{j}$$

---

② MKS



$I_{K1} = I_g = 10A$ (1)

$+R_{21}I_{K1} + R_{22}I_{K2} + R_{23}I_{K3} = \mathcal{E}E_{K2}$ (2)

$-R_2 I_g + (R_2+R_4) I_{K2} + 0 = -E$

$8 I_{K2} = -20 + 40 = 20$

$$I_{K2} = \frac{20}{8} = \frac{5}{2} = 2,5A$$

$R_{31}I_{K1} + R_{32}I_{K2} + R_{33}I_{K3} = \mathcal{E}E_{K3}$ (3)

$-R_1 I_g + 0 + (R_1+R_3) I_{K3} = E$

$4 I_{K3} = 20 + 20 = 40$

$$I_{K3} = \frac{40}{4} = 10A$$

$I_4 = I_{K2} = 2,5A$

$P_{R4} = R_4 \cdot I_4^2 = 4 \cdot 6,25 = 25W$

$I_3 = I_{K3} = 10A$

$U_{BA} = R_4 I_4 + R_3 I_3 = 4 \cdot 2,5 + 2 \cdot 10 = 30V$

$P_{Ig} = U_{BA} \cdot I_g = 300W$

MPC: $V_D = 0$

$V_A\left(\frac{1}{R_1}+\frac{1}{R_3}\right) - \frac{1}{R_3}V_C = -I_g$

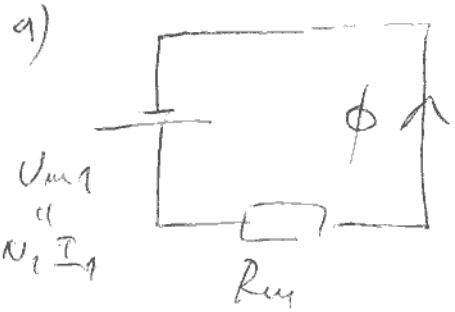
$V_B\left(\frac{1}{R_2}+\frac{1}{R_4}\right) - \frac{1}{R_4}V_C = I_g$

$V_C = E$

$\Rightarrow V_A = 0, V_B = 30V$

---

③ a)



$$R_m = \frac{\ell}{\mu S} = \frac{50\cdot 10^{-2}}{2\cdot 10^{-4}\, 10 (10^{-2})^2} = 2,5\cdot 10^6 \frac{1}{H}$$

$$\phi = \frac{N_1 I_1}{R_m} = \frac{50\cdot 12}{2,5\cdot 10^6} = 240\,\mu Wb$$

$$B = \frac{\phi}{S} = \frac{240\cdot 10^{-6}}{10\cdot 10^{-4}} = 0,24\,T$$

$$H = \frac{B}{\mu} = \frac{0,24}{2\cdot 10^{-4}} = 1,2\cdot 10^3 A/m = 1200\,A/m$$

b) $w = \frac{1}{2} B \cdot H$

$W = w \cdot l \cdot S = \frac{1}{2} BH \cdot l \cdot S = \frac{1}{2} \cdot 0{,}24 \cdot 1200 \cdot 50 \cdot 10^{-2} \cdot 10 \cdot 10^{-4}$

$W = 72 \cdot 10^{-3}\ J = 72\ mJ$

c) $e_{ind} = -\frac{d(\phi \cdot N_2)}{dt} = 0$ (Fluks je konstantan u vremenu!)

---

(4)

a) $\bar{Z}_{L1} = j\omega L_1 = j\,500 \cdot 50 \cdot 10^{-3} = j25\,\Omega$

$\bar{Z}_{C1} = -j\frac{1}{\omega C_1} = -j\frac{1}{500 \cdot 40 \cdot 10^{-6}} = -j50\,\Omega$

$\Rightarrow \bar{Z}_1 = \bar{Z}_{L1} + \bar{Z}_{C1} = -j25\,\Omega$

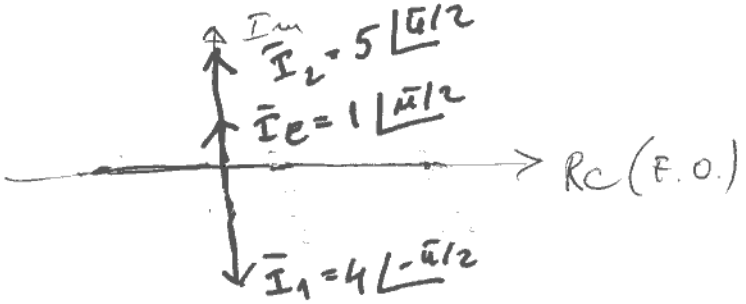
$\bar{Z}_{L2} = j\omega L_2 = j\,500 \cdot 40\,10^{-3} = j20\,\Omega \Rightarrow \bar{Z}_2 = \bar{Z}_{L2} = j20\,\Omega$

$\bar{I}_1 = \frac{\bar{E}}{\bar{Z}_1} = \frac{-100}{-j25} = -j4\,A$ $\quad \bar{I}_2 = \frac{\bar{E}}{\bar{Z}_2} = \frac{-100}{j20} = j5\,A$

$\bar{E} = 100 e^{-j\bar{u}} = -100\,V$

$\bar{I}_e = \bar{I}_1 + \bar{I}_2$

$\bar{I}_e = j\,A$



b) $\bar{U}_{C1} = \bar{Z}_{C1}\bar{I}_1 = (-j50)(-j4) = -200\,V = 200 e^{-j\bar{u}}\,V$

$u_{C1}(t) = 200\sqrt{2}\cos(500t - \bar{u})\,V$

c) $\bar{S}_e = \bar{E} \cdot \bar{I}_e^* = (-100)\cdot(-j) = j100\,VA$

$P_e = 0\,W,\ Q_e = 100\,VAR,\ S_e = 100\,VA,\ \cos\varphi = \frac{P_e}{S_e} = 0$

d) $\bar{Y}_1 = \frac{1}{\bar{Z}_1} = \frac{1}{-j25} = j40\,mS$

$\bar{Y}_2 = \frac{1}{\bar{Z}_2} = \frac{1}{j20} = -j50\,mS$

$\bar{Y}_e = \bar{Y}_1 + \bar{Y}_2 = -j10\,mS$

(PARALELNA VEZA!)

$\bar{Z}_e = \bar{Z}_1 \| \bar{Z}_2 = \frac{\bar{Z}_1\bar{Z}_2}{\bar{Z}_1 + \bar{Z}_2} = \frac{(-j25)j20}{-j25 + j20}$

$\bar{Z}_e = \frac{500}{-j5} = j100\,\Omega$

$\bar{Y}_e = \frac{1}{\bar{Z}_e} = \frac{1}{j100} = -j10\,mS$

(5)

$U_L = \sqrt{3}E$ (Y GENERATOR)

$U_L = 500\sqrt{3}\ V$

$U_F = U_L = 500\sqrt{3}\ V$ (Δ POTROŠAČ)

$Z_F = \frac{U_F}{I_F} = \frac{500\sqrt{3}}{\sqrt{3}} = 500\ \Omega$

$S = 3U_F I_F \Rightarrow \cos\varphi = \frac{P}{S} = \frac{P}{3U_F I_F} = \frac{1700}{3 \cdot 500\sqrt{3} \cdot \sqrt{3}} = 0{,}6 = \frac{3}{5}$

$P = S\cos\varphi$

$\sin\varphi = -\sqrt{1-\cos^2\varphi} = -\sqrt{1-\left(\frac{3}{5}\right)^2} = -\sqrt{1-\frac{9}{25}}$

$\sin\varphi = -\sqrt{\frac{16}{25}} = -\frac{4}{5} = -0{,}8$ (<0 KAPAC.)

$\bar{Z}_F = Z_F(\cos\varphi + j\sin\varphi) = 500\left(\frac{3}{5} - j\frac{4}{5}\right) = (300 - j400)\ \Omega$