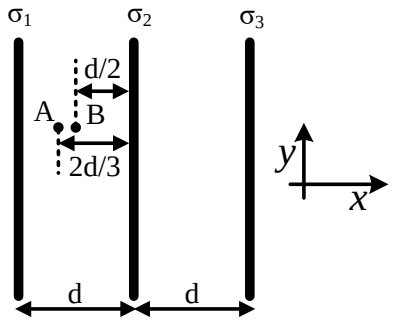
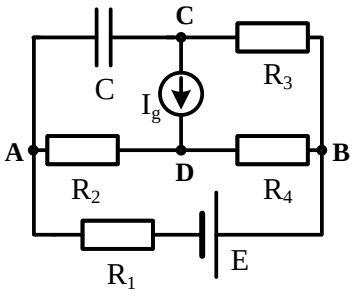
Elektrotehnika - pismeni ispit

7. jul 2026.

1. Tri veoma tanke i veoma velike ravnomerno naelektrisane metalne ploče nalaze se u sredini čija je dielektrična konstanta $\varepsilon = 0.5 \cdot 10^{-8}\ \mathrm{F/m}$, na međusobnom rastojanju od $d = 5\ \mathrm{cm}$ kao što je prikazano na Slici 1. Poznate su površinske gustine naelektrisanja ploča $\sigma_1 = 80\ \mathrm{nC/m^2}$, $\sigma_2 = -60\ \mathrm{nC/m^2}$ i $\sigma_3 = 20\ \mathrm{nC/m^2}$. Odrediti vektore električnog polja u tačkama A i B, a zatim odrediti napon U_{AB}. (**10 poena**)



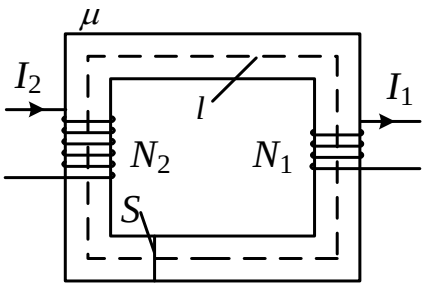
Slika 1



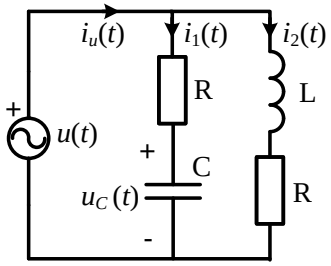
Slika 2

2. U kolu sa Slike 2 potrebno je odrediti energiju kondenzatora C, snagu strujnog generatora I_g i snagu idealnog naponskog generatora E.
Poznato je: $E = 40\ V$, $I_g = 4\ A$, $R_1 = R_2 = R = 10\ \Omega$, $R_3 = R_4 = 2R = 20\ \Omega$, $C = 2$ nF. **(12 poena)**

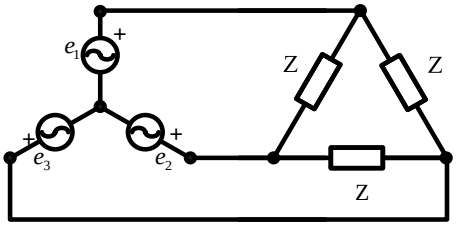
3. U kolu na Slici 3 prikazano je magnetno kolo sa dva namotaja. Kroz namotaj sa $N_1 = 100$ navojaka protiče struja $I_1 = 3\ A$, a kroz namotaj sa $N_2 = 500$ navojaka protiče struja $I_2 = 1\ A$. Jezgro je površine poprečnog preseka $S = 10\ \mathrm{cm^2}$, dužine srednje linije $l = 40\ \mathrm{cm}$ i magnetne permeabilnosti $\mu = 2 \cdot 10^{-5}\ \mathrm{H/m}$.
a) Odrediti fluks vektora magnetne indukcije i jačinu magnetnog polja u jezgru. **(6 poena)**
b) Odrediti koju vrednost bi trebalo da ima struja u drugom namotaju, $I_2' = ?$, tako da energija magnetnog polja u jezgru bude jednaka nuli. **(2 poena)**



Slika 3



Slika 4



Slika 5

4. Na Slici 4 je prikazano kolo naizmenične struje poznatih parametara: $R = 50\ \Omega$, $L = 100\ \mathrm{mH}$, $C = 40\ \mu\mathrm{F}$ i $u(t) = 200\sqrt{2}\cos(500t)\ \mathrm{V}$. Odrediti:
a) kompleksne izraze za sve označene struje u kolu i skicirati njihov fazorski dijagram, **(6 poena)**
b) vremenski oblik napona na kondenzatoru, **(2 poena)**
c) kompleksnu impedansu celokupnog potrošača, **(1 poen)**
d) aktivnu, reaktivnu i prividnu snagu celokupnog potrošača. **(3 poena)**

5. Na Slici 5 prikazan je trofazni sistem generator-potrošač. Efektivna vrednost elektromotornih sila generatora iznosi $E = 400\ V$, reaktivna snaga trofaznog potrošača $Q = -12\ \mathrm{kVAr}$, a faktor snage je $\sqrt{3}/2$. Odrediti linijsku struju i kompleksnu impedansu potrošača. **(8 poena)**

(1)

$$\vec{K} = \vec{K}_A = \vec{K}_B$$

$$\vec{K} = \frac{\sigma_1}{2\varepsilon}\vec{i} + \frac{\sigma_2}{2\varepsilon}(-\vec{i}) + \frac{\sigma_3}{2\varepsilon}(-\vec{i}) = \frac{\sigma_1 - \sigma_2 - \sigma_3}{2\varepsilon}\vec{i}$$

$$\vec{K} = \frac{80\mu - (-60\mu) - 20\mu}{2 \cdot 0{,}5 \cdot 10^{-8}}\vec{i} = \frac{120 \cdot 10^{-9}}{10^{-8}}\vec{i} = 12\,\vec{i}\ \frac{V}{m}$$

$$U_{AB} = \int_A^B \vec{K}\,d\vec{l} = \vec{K} \cdot \Delta\vec{l} = \vec{K} \cdot \left(\frac{d}{6}\vec{i}\right) = 12\,\vec{i} \cdot \frac{5 \cdot 10^{-2}}{6} \cdot \vec{i} = 0{,}1\,V$$

$$\Delta l = \frac{2d}{3} - \frac{d}{2} = \frac{4d - 3d}{6} = \frac{d}{6}$$

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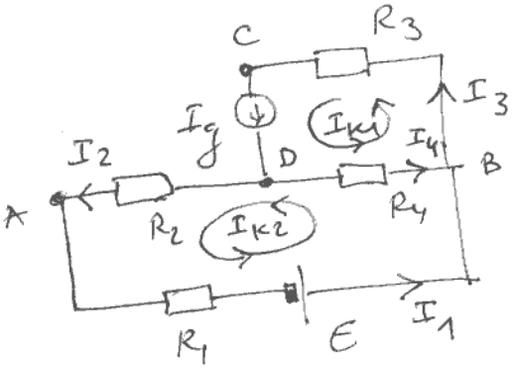
(2)



MKS: $I_{K1} = I_g = 4A$

$$R_{21} I_{K1} + R_{22} I_{K2} = \mathcal{E}_{K2}$$

$$-R_4 I_g + (R_1 + R_2 + R_4) I_{K2} = E$$

$$I_{K2} = \frac{E + R_4 I_g}{R_1 + R_2 + R_4} = \frac{E_1 + 2RI_g}{4R}$$

$$I_{K2} = \frac{E}{4R} + \frac{I_g}{2} = \frac{40}{40} + \frac{4}{2} = 3A$$

$I_1 = I_{K2} = 3A$

$I_2 = I_1 = I_{K2} = 3A$

$I_3 = I_g = 4A$

$I_4 = I_{K1} - I_{K2} = 1A$

$U_{DC} = R_4 I_4 + R_3 I_3 = 100V$

$\boxed{P_{Ig} = U_{DC} \cdot I_g = 400W}$

$U_{AC} = -R_2 I_2 + R_3 I_3 + R_4 I_4 = -10 \cdot 3 + 20 \cdot 4 + 20 \cdot 1$

$U_{AC} = 70\ V \Rightarrow \boxed{W_C = \frac{1}{2} C U_{AC}^2 = \frac{1}{2} \cdot 2 \cdot 10^{-9} \cdot 70^2}$

$\boxed{W_C = 4{,}9\,\mu J}$

$\boxed{P_E = E \cdot I_1 = 120W}$

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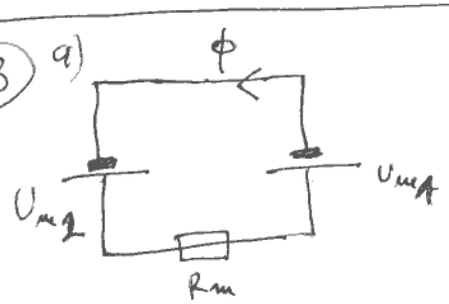
(3) a)



$$R_m = \frac{l}{\mu S},\quad U_{m1} = N_1 I_1,\quad U_{m2} = N_2 I_2$$

$$\phi = \frac{U_{m2} - U_{m1}}{R_m} = \frac{N_2 I_2 - N_1 I_1}{\frac{l}{\mu S}} = \frac{500 \cdot 1 - 100 \cdot 3}{\frac{40 \cdot 10^{-2}}{2 \cdot 10^{-5} \cdot 10 \cdot 10^{-4}}}$$

$$\phi = 10^{-5}\,Wb = 10\,\mu Wb$$

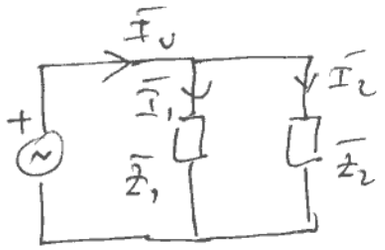
$$B = \frac{\phi}{S} = \frac{10 \cdot 10^{-6}}{10 \cdot 10^{-4}} = 10^{-2}\,T = 0{,}01\,T$$

$$H = \frac{B}{\mu} = \frac{10^{-2}}{2 \cdot 10^{-5}} = 500\ A/m$$

b) $W_m = 0 \Rightarrow B, H = 0 \Rightarrow \phi = 0 \Rightarrow N_2 I_2' - N_1 I_1 = 0 \Rightarrow I_2' = \frac{N_1 I_1}{N_2} = \frac{3}{5} = 0{,}6A$

(4)

a)



$\bar{U} = 200 e^{j0} = 200\,V$

$\bar{Z}_L = j\omega L = j500 \cdot 100 \cdot 10^{-3} = j50\,\Omega$

$\bar{Z}_2 = R + \bar{Z}_L = (50 + j50)\,\Omega = 50\sqrt{2}\, e^{j\pi/4}\,\Omega$

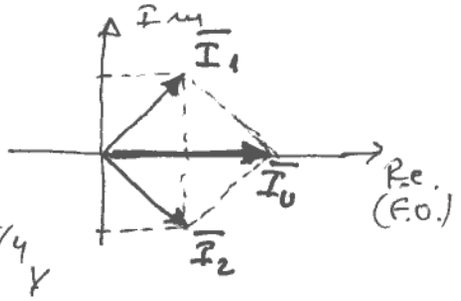
$\bar{Z}_C = -j\frac{1}{\omega C} = -j\frac{1}{500 \cdot 40 \cdot 10^{-6}} = -j50\,\Omega$

$\bar{Z}_1 = R + \bar{Z}_C = (50 - j50)\,\Omega = 50\sqrt{2}\, e^{-j\pi/4}\,\Omega$

$\bar{I}_1 = \frac{\bar{U}}{\bar{Z}_1} = \frac{200}{50\sqrt{2}\, e^{-j\pi/4}} = \frac{4}{\sqrt{2}} e^{+j\pi/4} = 2\sqrt{2}\, e^{j\pi/4}\,A = (2 + j2)\,A$

$\bar{I}_2 = \frac{\bar{U}}{\bar{Z}_2} = \frac{200}{50\sqrt{2}\, e^{j\pi/4}} = 2\sqrt{2} e^{-j\pi/4}\,A = (2 - j2)\,A$

$\bar{I}_U = \bar{I}_1 + \bar{I}_2 = 4\,A$



b) $\bar{U}_C = \bar{Z}_C \cdot \bar{I}_1 = 50 e^{-j\pi/2} \cdot 2\sqrt{2}\, e^{j\pi/4} = 100\sqrt{2}\, e^{-j\pi/4}\,V$

$u_C(t) = 200 \cos(500t - \pi/4)\,V$

c) $\bar{Z}_e = \frac{\bar{Z}_1 \cdot \bar{Z}_2}{\bar{Z}_1 + \bar{Z}_2} = \frac{50\sqrt{2}\, e^{j\pi/4} \cdot 50\sqrt{2}\, e^{-j\pi/4}}{50 + j50 + 50 - j50} = \frac{2500 \cdot 2}{50 \cdot 2} = 50\,\Omega$

(ili $\bar{Z}_e = \frac{\bar{U}}{\bar{I}_U} = \frac{200}{4} = 50\,\Omega$)

d) $\bar{S} = \bar{U}\,\bar{I}_U^{*} = \bar{Z}_e I_U^2 = 50 \cdot 16 = (800 + j0)\,VA$

$P = 800\,W$, $Q = 0\,VAR$, $S = 800\,VA$

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(5) GEN Y: $U_L = U\sqrt{3} = 400\sqrt{3}\,V$

POT Δ: $U_F = U_L = 400\sqrt{3}\,V$

$\cos\varphi = \sqrt{3}/2$

$\sin\varphi = -\sqrt{1 - \cos^2\varphi} = -\sqrt{1 - \frac{3}{4}} = -1/2$

↑ KAPACITIVNI JER JE $Q < 0$

$Q = 3U_F I_F \sin\varphi \Rightarrow I_F = \frac{Q}{3U_F \sin\varphi} = \frac{-12000}{3 \cdot 400\sqrt{3} \cdot (-1/2)} = \frac{20}{\sqrt{3}}\,A$

$\boxed{I_L = I_F\sqrt{3} = 20\,A}$

$Z = \frac{U_F}{I_F} = \frac{400\sqrt{3}}{\frac{20}{\sqrt{3}}} = 60\,\Omega$

$\bar{Z} = Z\cos\varphi + jZ\sin\varphi = Z e^{j\varphi}$

$\bar{Z} = (30\sqrt{3} - j30)\,\Omega$