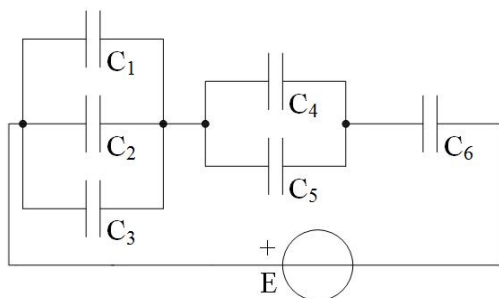


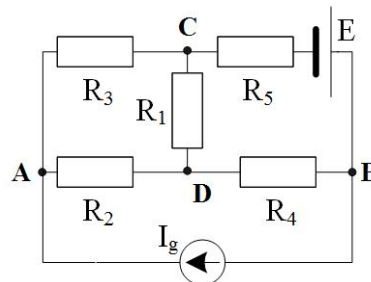
Elektrotehnika - pismeni ispit

2. septembar 2026.

1. U kolu na Slici 1 poznate su kapacitivnosti $C_1 = C_2 = C_3 = C_4 = C = 6 \mu\text{F}$, $C_5 = 2C$, $C_6 = 3C$ i elektromotorna sila idealnog naponskog generatora $E = 15 \text{ V}$. Pre povezivanja, svi kondenzatori su bili neopterećeni. Odrediti napon na C_6 , naelektrisanje na C_4 i energiju kondenzatora C_3 . (8 poena)



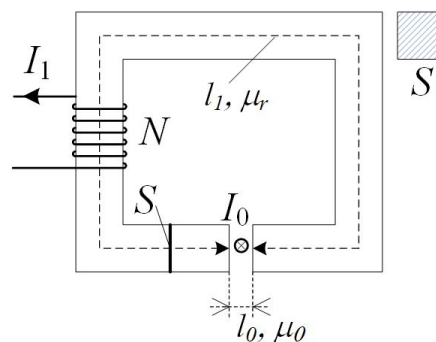
Slika 1



Slika 2

2. U kolu sa Slike 2 potrebno je, primenom Tevenenove teoreme, odrediti intenzitet struje, napon i snagu otpornika R_3 . Poznato je: $E = 40 \text{ V}$, $I_g = 8 \text{ A}$, $R_1 = R_4 = 20 \Omega$, $R_2 = R_5 = 10 \Omega$, $R_3 = 3 \Omega$. (12 poena)

3. Na pravougaono jezgro od feromagnetnog materijala, relativne magnetne permeabilnosti $\mu_r = 2000$, sa vazдушnim procepom širine $l_0 = 2 \text{ mm}$, ravnomerno je namotano $N = 20000$ navojaka, tako da nema magnetnog rasipanja (Slika 3). Dužina srednje linije jezgra iznosi $l_1 = 20 \text{ cm}$, a površina poprečnog preseka jezgra, kvadratnog oblika, iznosi $S = 9 \text{ cm}^2$. Sila koja deluje na tanak pravolinijski provodnik, postavljen paralelno ivicama preseka, kroz koji protiče struja intenziteta $I_0 = 1 \text{ A}$, iznosi $F = 3 \cdot 10^{-2} \text{ N}$. Odrediti: intenzitet vektora magnetne indukcije u procepu, fluks u jezgru i gustinu energije magnetnog polja u feromagnetnom delu jezgra. (10 poena)

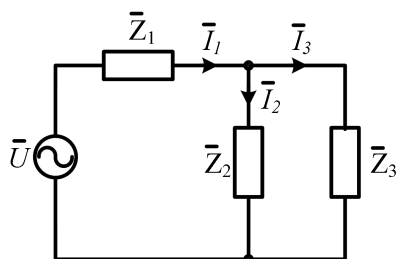


Slika 3

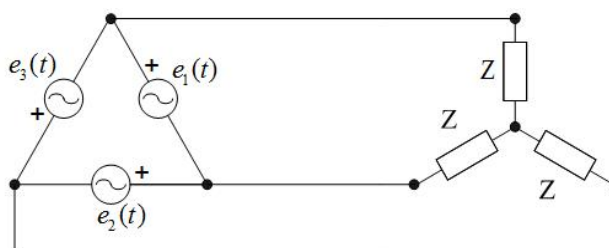
4. Na Slici 4 je prikazano kolo naizmenične struje koje čine naponski generator efektivne vrednosti napona $U = 100\sqrt{2} \text{ V}$ i tri potrošača kompleksnih impedansi: $\bar{Z}_1 = (10 + j20) \Omega$, $\bar{Z}_2 = (10 - j30) \Omega$ i $\bar{Z}_3 = (10 + j10) \Omega$. Odrediti:

a) kompleksne izraze za struje u svim granama kola; (6 poena)

b) reaktivnu snagu potrošača Z_1 i kompleksnu prividnu snagu potrošača Z_2 . (6 poena)



Slika 4



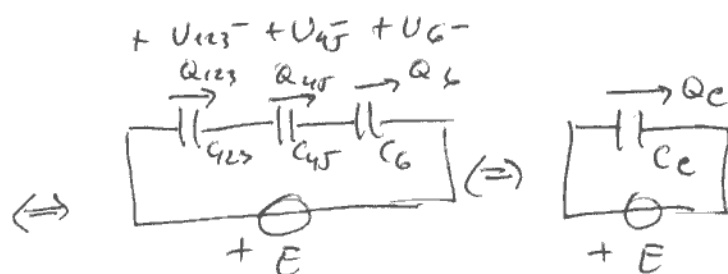
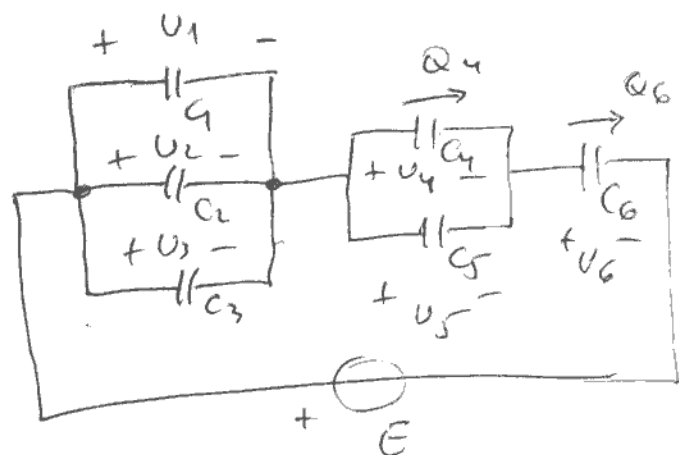
Slika 5

5. Na Slici 5 je prikazan simetričan trofazni sistem generator - potrošač. Ems generatora iznose: $e_1(t) = 400 \cdot \cos(314t + \pi/2) \text{ V}$, $e_2(t) = 400 \cdot \cos(314t + 7\pi/6) \text{ V}$, $e_3(t) = 400 \cdot \cos(314t - \pi/6) \text{ V}$, dok impedansa jedne faze potrošača iznosi $\bar{Z} = 20 - j10 \Omega$.

a) Nacrtati fazorski dijagram linijskih napona. (2 poena)

b) Odrediti efektivnu vrednost linijskih struja, prividnu i reaktivnu snagu potrošača. (6 poena)

①



$$C_{123} = C_1 + C_2 + C_3 = 3C = 18\mu F$$

$$C_{45} = C_4 + C_5 = C + 2C = 3C = 18\mu F$$

$$C_6 = 3C = 18\mu F$$

$$C_e = \frac{1}{\frac{1}{C_{123}} + \frac{1}{C_{45}} + \frac{1}{C_6}} = \frac{1}{\frac{3}{3C}} = C = 6\mu F$$

$$Q_e = C_e \cdot E = CE = 6 \cdot 15 = 90\mu C$$

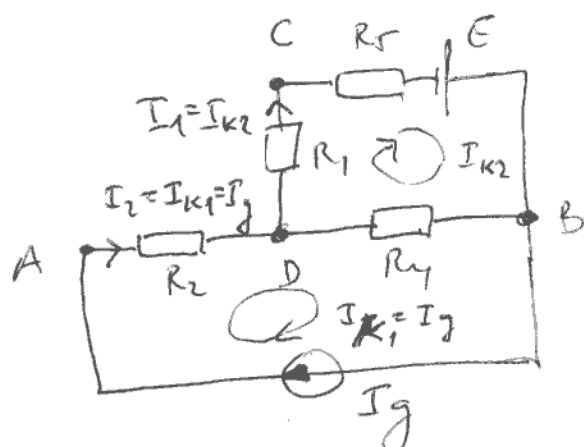
$$Q_{123} = Q_{45} = Q_6 = Q_e = CE = 90\mu C \Rightarrow U_6 = \frac{Q_6}{C_6} = \frac{CE}{3C} = \frac{E}{3} = 5V$$

$$U_4 = U_5 = \frac{Q_{45}}{C_{45}} = \frac{CE}{3C} = \frac{E}{3} = 5V \Rightarrow Q_4 = C_4 U_4 = \frac{CE}{3} = 30\mu C$$

$$U_1 = U_2 = U_3 = U_{123} = \frac{Q_{123}}{C_{123}} = \frac{CE}{3C} = \frac{E}{3} = 5V$$

$$W_{C3} = \frac{1}{2} C_3 U_3^2 = \frac{1}{2} C \cdot \left(\frac{E}{3}\right)^2 = \frac{CE^2}{18} = \frac{6 \cdot 15 \cdot 15}{18} = 75\mu J$$

②



$$E_T = U_{AC}$$

$$\text{MKS: } I_{K1} = I_g = 8A$$

(1)

$$(-R_1) I_{K1} + (R_1 + R_4 + R_5) I_{K2} = E \quad (2)$$

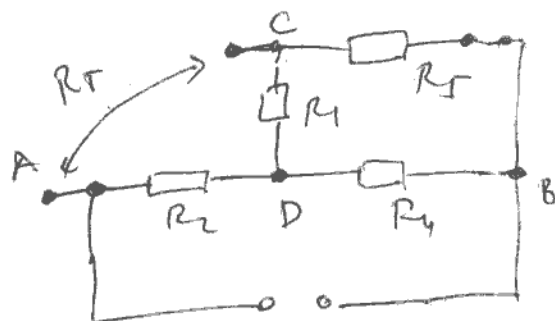
$$I_{K2} = \frac{E + R_1 I_g}{R_1 + R_4 + R_5} = \frac{40 + 20 \cdot 8}{10 + 20 + 10} = 4A$$

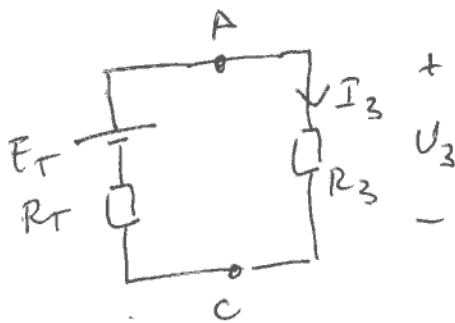
$$I_A = I_{K2} = 4A$$

$$E_T = U_{AC} = R_2 I_2 + R_1 I_1 = 10 \cdot 8 + 10 \cdot 4 = 160V$$

$$R_T = R_{AC} = R_2 + R_4 \parallel (R_1 + R_5)$$

$$R_T = 10 + \frac{1}{\frac{1}{20} + \frac{1}{30}} = 10 + \frac{60}{3+2} = 22\Omega$$





$$I_3 = \frac{E_T}{R_T + R_3} = \frac{160}{22 + 3} = \frac{160}{25} = 6,4 \text{ A}$$

$$U_3 = R_3 I_3 = 3 \cdot 6,4 = 19,2 \text{ V}$$

$$P_3 = U_3 I_3 = R_3 I_3^2 = 3 \cdot \left(\frac{64}{10}\right)^2 = \frac{3 \cdot 4096}{100} = 122,88 \text{ W}$$

③ $|\vec{F}| = F = I_0 l B$, $l = \sqrt{5} \Rightarrow B_0 = \frac{F}{I_0 \sqrt{5}} = \frac{3 \cdot 10^{-2}}{1 \cdot \sqrt{5 \cdot 10^{-4}}} = \frac{3 \cdot 10^{-2}}{3 \cdot 10^{-2}} = 1 \text{ T}$

$$B \approx B_0 = 1 \text{ T}$$

$$\Phi = BS = 1 \cdot 9 \cdot 10^{-7} = 9 \cdot 10^{-7} \text{ Wb} = 0,9 \mu\text{Wb}$$

$$W = \frac{1}{2} B \cdot H = \frac{1}{2} B \cdot \frac{B}{\mu_0 \mu_r} = \frac{B^2}{2 \mu_0 \mu_r} = \frac{1}{2 \cdot 4\pi \cdot 10^{-7} \cdot 2000} = \frac{10^7}{16\pi \cdot 10^3}$$

$$w = \frac{10^4}{16\pi} = \frac{10^5 \cdot 10^5 \cdot 10^5 \cdot 10^5}{4 \cdot 2 \cdot 2 \cdot 2 \cdot \pi} = \frac{625}{\pi} \frac{\text{J}}{\text{m}^3}$$

④ a) $\bar{z}_{23} = \bar{z}_2 \parallel \bar{z}_3 = \frac{\bar{z}_2 \bar{z}_3}{\bar{z}_2 + \bar{z}_3} = \frac{(10 - j30)(10 + j10)}{10 + j10 + 10 - j30} = \frac{100 + 300 - j300 + j100}{20 - j20}$

$$\bar{z}_{23} = \frac{400 - j200}{20 - j20} = \frac{20 - j10}{1 - j} \cdot \frac{1 + j}{1 + j} = \frac{20 - j10 + 20j + 10}{2} = (15 + j5) \Omega$$

$$\bar{U} = U e^{j0} = 100\sqrt{2} \text{ V (usvojeno početne fazu 0)}$$

$$\bar{I}_1 = \frac{\bar{U}}{\bar{z}_1 + \bar{z}_{23}} = \frac{100\sqrt{2}}{10 + j20 + 15 + j5} = \frac{100\sqrt{2}}{25 + j25} = \frac{4\sqrt{2}}{1 + j} \cdot \frac{1 - j}{1 - j} = \frac{4\sqrt{2}(1 - j)}{2}$$

$$\bar{I}_1 = (2\sqrt{2} - j2\sqrt{2}) \text{ A} \Rightarrow I_1 = \sqrt{(2\sqrt{2})^2 + (2\sqrt{2})^2} = 4 \text{ A}$$

$$\bar{U}_{23} = \bar{z}_{23} \cdot \bar{I}_1 = (15 + j5)(2\sqrt{2} - j2\sqrt{2}) = 30\sqrt{2} - j30\sqrt{2} + j10\sqrt{2} + 10\sqrt{2}$$

$$\bar{U}_{23} = (40\sqrt{2} - j20\sqrt{2}) \text{ V}$$

$$\bar{I}_2 = \frac{\bar{U}_{23}}{\bar{z}_2} = \frac{40\sqrt{2} - j20\sqrt{2}}{10 - j30} = \frac{4\sqrt{2} - j2\sqrt{2}}{1 - j3} \cdot \frac{1 + j3}{1 + j3} = \frac{4\sqrt{2} - j2\sqrt{2} + 12\sqrt{2}j + 6\sqrt{2}}{1 + 9}$$

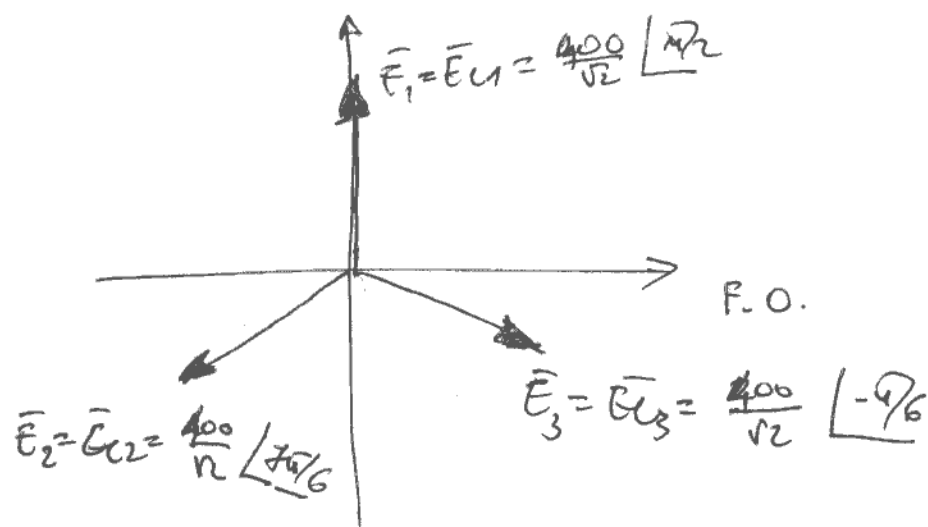
$$\bar{I}_2 = (\sqrt{2} + j\sqrt{2}) \text{ A}$$

$$\bar{I}_3 = \bar{I}_1 - \bar{I}_2 = 2\sqrt{2} - j2\sqrt{2} - (\sqrt{2} + j\sqrt{2}) = (\sqrt{2} - j3\sqrt{2}) \text{ A}$$

$$\bar{S}_2 = \bar{U}_{23} \cdot \bar{I}_2^* = (40\sqrt{2} - j20\sqrt{2})(\sqrt{2} - j\sqrt{2}) = 80 - j40 - j80 - 40 = (40 - j120) \text{ VA}$$

$$Q_1 = jX_1 \cdot I_1^2 = j10 \cdot (4)^2 = j320 \text{ VAR}$$

5) a)



b) Generator je povezan u Δ :

$$U_L = E = \frac{E_m}{\sqrt{2}} = \frac{400}{\sqrt{2}} = 200\sqrt{2} \text{ V}$$

POTROŠAČ je vezan u Y:

$$U_F = \frac{U_L}{\sqrt{3}} = \frac{200\sqrt{2}}{\sqrt{3}} \text{ V}$$

$$Z = \sqrt{20^2 + (-10)^2} = \sqrt{10^2(2^2 + 1)} = 10\sqrt{5} \Omega$$

$$I_F = \frac{U_F}{Z} = \frac{200\sqrt{2}}{\sqrt{3} \cdot 10\sqrt{5}} = 20\sqrt{\frac{2}{15}} \text{ A}$$

$$I_L = I_F = 20\sqrt{\frac{2}{15}} \text{ A}$$

$$S = 3 Z I_F^2 = 3 \cdot 10\sqrt{5} \cdot 400 \cdot \frac{2}{15} = 1600\sqrt{5} \text{ VA}$$

$$Q = 3 X I_F^2 = 3 \cdot (-10) \cdot 400 \cdot \frac{2}{15} = -1600 \text{ VAR}$$