1) a) $\int \frac{2\sin 2x\,dx}{(1+\sin^2 x)^3} = \left[\begin{array}{c} \text{смена: } 1+\sin^2 x = t \\ d(1+\sin^2 x) = dt \\ \sin 2x\,dx = dt \end{array}\right] =$

$= \int 2t^{-3}\,dt = C - \frac{1}{t^2} = C - \frac{1}{(1+\sin^2 x)^2}$

б) $\int \frac{\operatorname{arctg} x \ \ln(\operatorname{arctg} x)}{x^2+1}\,dx = \left[\begin{array}{c} \text{смена:} \\ \operatorname{arctg} x = t \\ \frac{dx}{x^2+1} = dt \end{array}\right] =$

$= \int t \ln t \overset{\text{пи}}{=} \int \ln t \, d\left(\frac{t^2}{2}\right) = \frac{t^2}{2}\ln t - \int \frac{t^2}{2}\, d(\ln t)$

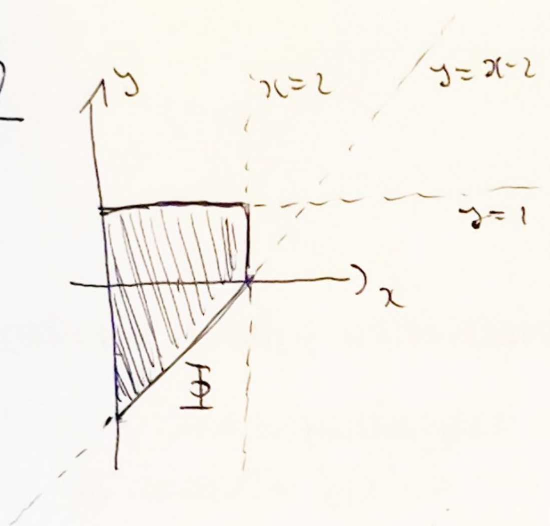
$= \frac{t^2}{2}\ln t - \frac{1}{2}\int t\,dt = \frac{1}{4}t^2(2\ln t - 1) + C =$

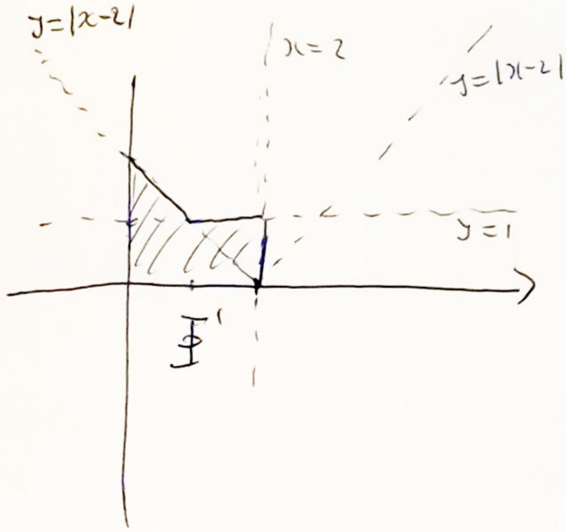
$= \operatorname{arctg}^2 x \ \frac{\ln(\operatorname{arctg} x)^2 - 1}{4} + C$

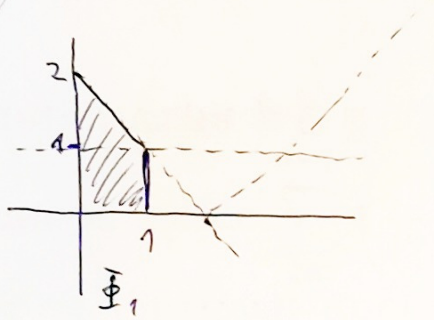
в) $\int \frac{x^2\,dx}{\sqrt[3]{x^3+3}} = \left[\begin{array}{cc} \text{смена:} & d(x^3+3) = d(t^3) \\ \sqrt[3]{x^3+3} = t & 3x^2 dx = 3t^2 dt \\ x^3+3 = t^3 & x^2 dx = t^2 dt \end{array}\right] =$

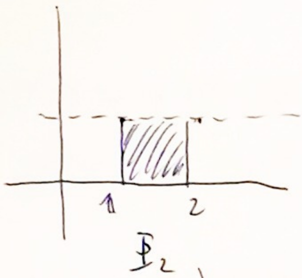
$= \int \frac{t^2\,dt}{t} = C + \frac{t^2}{2} = C + \frac{1}{2}\sqrt[3]{x^3+3}^{\,2}$

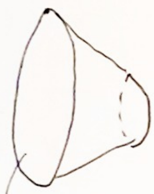
2











ЗАРУБЉЕНА КУПА

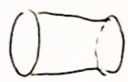
$H=1 \quad r_1=2 \quad r_2=1$

$V_1 = \frac{7\pi}{3}$

$M_1 = 3\pi\sqrt{2}$

$B_1 = 4\pi$

ВАЉАК



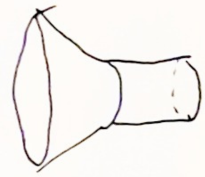
$H=1$

$r=1$

$V_2 = \pi$

$M_2 = 2\pi$

$B_2 = \pi$



$V = V_1 + V_2 = \frac{10\pi}{3}$

$P = B_1 + M_1 + M_2 + B_2$

$P = (7 + 3\sqrt{2})\pi$

ИЛИ

$$f(x) = \begin{cases} 2-x, & 0 \le x \le 1 \\ 1, & 1 \le x \le 2 \end{cases}$$

$$V = \pi \int_0^2 f^2(x)\,dx = \pi\left(\int_0^1 (2-x)^2 dx + \int_1^2 (1)^2 dx\right) = \frac{10\pi}{3}$$

$$M = 2\pi \int_0^2 f(x)\sqrt{2}\,dx = 2\pi\left(\sqrt{2}\int_0^1 (2-x)\,dx + \int_1^2 1\,dx\right) =$$

$$= -\pi \frac{(x-2)^2}{\cancel{2}}\sqrt{2}\Big|_0^1 + 2\pi x\Big|_1^2 = 3\pi\sqrt{2} + 2\pi = (3\sqrt{2}+2)\pi$$

$B_1 = r_1^2\pi = 4\pi$

$B_2 = r_2^2\pi = \pi$

$P = B_1 + M + B_2 = (3\sqrt{2}+7)\pi$

3. $y' = \dfrac{\sqrt{\cos 2x}}{\sin x}$ $\qquad 1 + y'^2 = 1 + \dfrac{\cos 2x}{\sin^2 x}$

$$= \frac{\sin^2 x + \cos^2 x - \sin^2 x}{\sin^2 x}$$

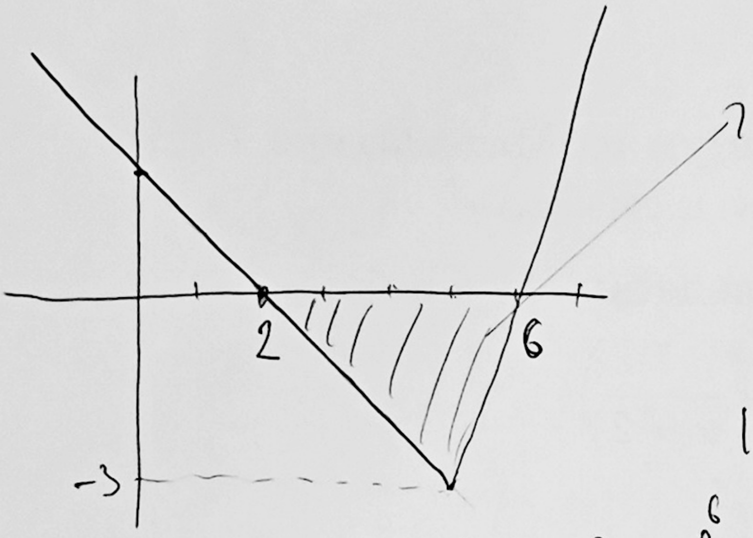
$$l = \int_{\pi/4}^{3\pi/4} \sqrt{1 + y'^2}\, dx$$

$$= \int_{\pi/4}^{3\pi/4} \frac{|\cos x|}{\sin x}\, dx = \int_{\pi/4}^{\pi/2} \frac{\cos x\, dx}{\sin x} + \int_{\pi/2}^{3\pi/4} \frac{-\cos x\, dx}{\sin x} =$$

$$= \ln|\sin x| \Big|_{\pi/4}^{\pi/2} - \ln|\sin x| \Big|_{\pi/2}^{3\pi/4} =$$

$$= \ln 1 - \ln \frac{1}{\sqrt{2}} - \left(\ln \frac{1}{\sqrt{2}} - \ln 1 \right) = 2 \ln \sqrt{2} = \ln 2$$

4) $y = |2x-10| + x - 8$



Троугао $a=4$, $h=3$

$\Downarrow$

$P = \frac{ah}{2} = 6$

или

$P = \int_2^6 (0-y)\,dx =$

$= -\int_2^5 |2x-10| + x - 8\,dx - \int_5^6 |2x-10| + x - 8\,dx =$

$= -\int_2^5 2-x\,dx - \int_5^6 3x-18\,dx =$

$= \left(\frac{x^2}{2} - 2x\right)\Big|_2^5 - 3\left(\frac{x^2}{2} - 6x\right)\Big|_5^6 = 6$

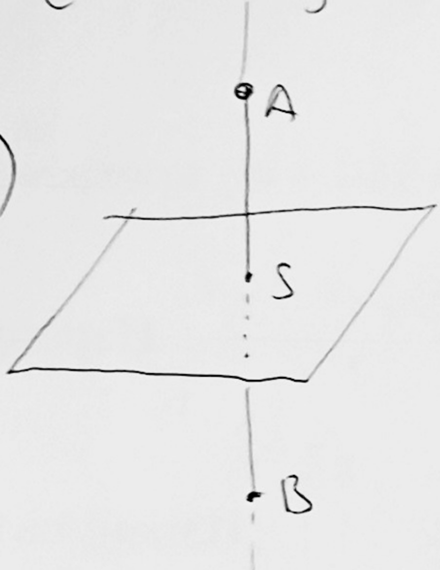
5 a) $p \cap q$: $\begin{cases} 2x + y - z = 2 \\ x + 2y \boxed{+z} = 4 \\ 3x + 3y - z = 2 \\ 5x + 4y - 2z = 4 \end{cases}$ ~

$\sim \begin{cases} x + 2y \boxed{+z} = 4 \\ 3x + 3y = 6 \\ 4x + 5y = 6 \\ 7x + 8y = 12 \end{cases} \sim \begin{cases} x + 2y \boxed{+z} = 4 \\ \boxed{x} + y = 2 \\ 4x + 5y = 6 \\ 7x + 8y = 12 \end{cases}$

$\sim \begin{cases} x + 2y \boxed{+z} = 4 \\ \boxed{x} + y = 2 \\ y = -2 \\ y = -2 \end{cases} \sim (x, y, z) = (4, -2, 4)$

$\boxed{A = (4, -2, 4)}$

б)



r: $A, B \in r$

$r \perp \alpha \Rightarrow v_r = n_\alpha = (1, 1, 1)$

$A \in r \Rightarrow r: \begin{cases} x = 4 + t \\ y = -2 + t \\ z = 4 + t \end{cases}$

$S \in r \cap \alpha \Rightarrow$ $4 + t + (-2 + t) + 4 + t = \ldots$

$3t + 6 = 0$

$t = -2$

$S = (2, -4, 2)$

$\overrightarrow{AS} = \overrightarrow{SB} \Rightarrow$ $B = (0, -6, 0)$

в) s: $N, B \in s$ $\qquad v_s = \overrightarrow{BN} = (1, 3, 1)$

$N \in p \cap s$

$B \in s$: $\begin{cases} x = 0 + t \\ y = -6 + 3t \\ z = 0 + t \end{cases}$

$2t - 6 + 3t - t = 2$ | $4t = 8$

$t - 12 + 6t + t = 4$ | $8t = 16$ $\quad t = 2 \quad N = (2, 0, 2)$

6) а) $P=\dfrac{1}{x+y}+e^{x}$ $\qquad Q=\dfrac{1}{x+y}+\cos y$

$$\left.\begin{aligned} P'_y&=\frac{-1}{(x+y)^2}\\ Q'_x&=\frac{-1}{(x+y)^2}\end{aligned}\right\} \Rightarrow \text{Постоји } z \text{ т.д. је } \begin{aligned} z'_x&=P \quad (1)\\ z'_y&=Q \quad (2)\end{aligned}$$

$$(1) \Rightarrow z=\varphi(y)+\int P\,dx=\varphi(y)+\int\frac{dx}{x+y}+\int e^{x}dx$$

$$\boxed{z=\varphi(y)+\ln|x+y|+e^{x}} \quad ☆$$

$$(2) \quad z'_y=Q$$
$$\Downarrow$$
$$\varphi'(y)+\frac{1}{x+y}+0=\frac{1}{x+y}+\cos y$$
$$!!$$
$$\varphi'(y)=\cos y \Rightarrow \varphi(y)=\int\cos y\,dy=\sin y$$

$$☆ \Rightarrow z=\ln|x+y|+e^{x}+\sin y$$

О.Р. $\boxed{\ln|x+y|+e^{x}+\sin y=C}$

б) Једначина нема С.Р.

7 a) $(xy' - y)(1+\cos^2 x) = y^2 \sin 2x$

$$\left(y' - \frac{y}{x}\right)(1+\cos^2 x) = \left(\frac{y}{x}\right)^2 x \sin 2x \quad \text{хомогена}$$

Смена
$\frac{y}{x} = z, \; z = z(x)$
$y' = z + xz'$

$$(z + xz' - z)(1+\cos^2 x) = z^2 x \sin 2x$$

$$x \cdot \frac{dz}{dx}(1+\cos^2 x) = z^2 x \sin 2x$$

$z = 0 \quad \vee \quad \frac{dz}{z^2} = \frac{2\sin x \cos x}{1+\cos^2 x} \; / (-1)$

Ј.К.З.С.Р.

$\Downarrow$

$\frac{y}{x} = 0$

$\boxed{y = 0}$

Ј.К.З.С.Р.

$$\frac{1}{z} = \int \frac{-2\sin x \cos x \, dx}{1+\cos^2 x}$$

$$\frac{x}{y} = c + \ln(1+\cos^2 x)$$

$$\boxed{y = \frac{x}{c + \ln(1+\cos^2 x)}}$$

Опште Решење

б) $y=0$ је једини кандидат за С.Р.

$C = \frac{1}{C_1} \Rightarrow \boxed{y = \frac{C_1 x}{1 + C_1 \ln(1+\cos^2 x)}}$

О.Р

За $C_1 = 0$ се из О.Р. добија $y = 0 \Rightarrow$

$y=0$ није С.Р.

Једначина нема С.Р.