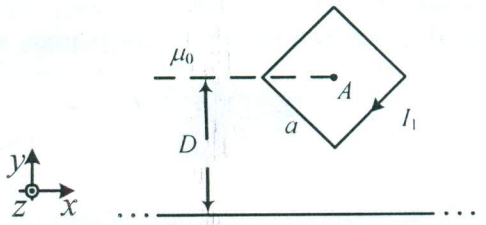
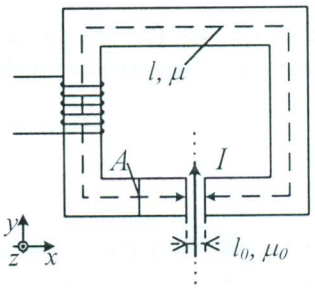
DRUGI KOLOKVIJUM IZ ELEKTROTEHNIKE
25. januar 2019.
GRUPA 1

1. Na Slici 1 prikazana je kontura oblika kvadrata stranice a=2m, kroz koju protiče struja intenziteta I_1=1A u označenom smeru. U istoj ravni, u vazduhu, nalazi se beskonačni pravolinijski provodnik kroz koji protiče nepoznata struja I_2. Odrediti smer i intenzitet struje I_2 tako da rezultujući vektor jačine magnetnog polja u tački **A** (na preseku dijagonala kvadrata) bude jednak nuli. Tačka A je udaljena $D=3a/2$=3m od pravolinijskog provodnika. **(8 poena)**

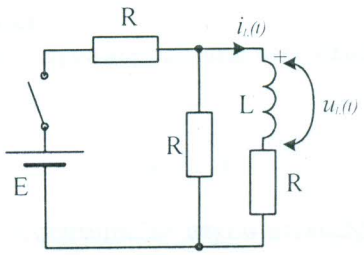
2. Na Slici 2 prikazano je magnetno kolo, koje se sastoji od jezgra, magnetne permeabilnosti $\mu = 10^{-8}$ H/m i dužine srednje linije $l = 50\text{cm}$. Poprečni presek jezgra je površine $A = 25\text{cm}^2$ i oblika kvadrata, a jezgro ima vazdušni procep debljine $l_0 = 1\text{mm}$. U procepu se nalazi pravolinijski provodnik, kroz koji protiče struja intenziteta $I = 10\text{A}$ u označenom smeru. Vektor magnetne indukcije u vazdušnom procepu je poznat i iznosi $\vec{B}_0 = 10^{-3} \cdot \vec{i}$ T.
a) Odrediti fluks vektora magn. indukcije i intenzitet vektora jačine magn. polja u jezgru magn. kola. **(6 poena)**
b) Odrediti i skicirati **vektor** sile kojom magnetno polje u procepu deluje na provodnik. **(6 poena)**



Slika 1



Slika 2

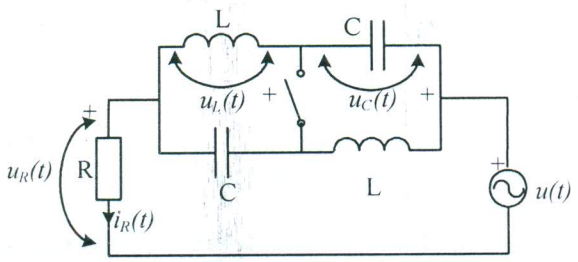


Slika 3

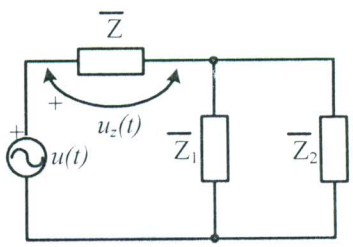
3. U kolu na Slici 3 poznate su parametri elemenata E, R i L. Prekidač П je otvoren i u kolu je uspostavljeno stacionarno stanje. U trenutku $t = 0$, prekidač se zatvara.
a) Odrediti izraz za struju i napon kalema nakon zatvaranja prekidača i nacrtati odgovarajuće vremenske dijagrame. **(7 poena)**
b) Odrediti intenzitet struje generatora u trenutku t_1=2L/R. **(3 poena)**

4. U kolu naizmenične struje na Slici 4 poznato je: $u(t) = 100\sqrt{2}\sin(\omega t)\,\text{V}$, $L = 1\text{mH}$, $C = 10\mu\text{F}$ i $R = 10\Omega$.
a) Prekidač je otvoren, a naponi na otporniku i generatoru su isti, tj. $u_R(t) = u(t)$. Odrediti kružnu učestanost generatora ω, kompleksne oblike struje kroz otpornik $\overline{I}_R$ i napona na kondenzatoru $\overline{U}_C$ i kalemu $\overline{U}_L$. **(8 poena)**
b) Ako se prekidač zatvori, odrediti kompleksni oblik struje kroz otpornik $\overline{I}_R$. **(5 poena)**

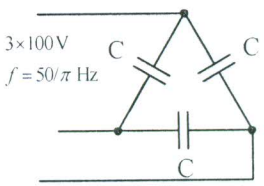
5. U kolu na Slici 5 poznato je: $u(t) = 200\sqrt{2}\sin(1000t)\,\text{V}$, $\overline{Z} = 5e^{j\pi/6}\Omega$, $\overline{Z}_1 = \overline{Z}_2 = 10e^{j\pi/6}\Omega$.
a) Izračunati efektivnu vrednost napona U_z na kompleksnoj impedansi $\overline{Z}$. **(4 poena)**
b) Odrediti aktivnu, reaktivnu i prividnu snagu na kompleksnoj impedansi $\overline{Z}_2$. **(6 poena)**



Slika 4



Slika 5

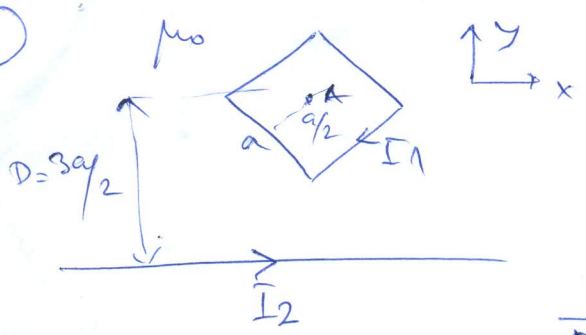


Slika 6

6. Na sistem trofaznog napona $3\times100\text{V}$, $f = 50/\pi$ Hz priključen je simetričan trofazni potrošač povezan u trougao, koji je sačinjen od tri kondenzatora kapacitivnosti $C = 100\,\mu\text{F}$ (Slika 6). Odrediti efektivnu vrednost linijske struje, aktivnu i prividnu snagu potrošača. **(7 poena)**

61

(1)



$$\vec{B}_{A\square} = 4 \cdot \frac{\mu_0 I_1}{4\pi \cdot a/2}\left(\cos 45 + \cos 45\right)(-\vec{k})$$

$$= \frac{2\mu_0 I_1}{\pi a}\left(\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}\right)(-\vec{k})$$

$$= \frac{-2\mu_0\sqrt{2} I_1}{\pi a}\vec{k}$$

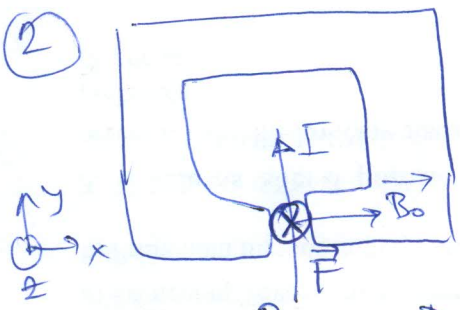
$$\vec{B}_{Ap} = \frac{\mu_0 I_2}{2\pi D}\vec{k} = \frac{\mu_0 I_2}{2\pi \frac{3a}{2}}\vec{k} = \frac{\mu_0 I_2}{3\pi a}\vec{k}$$

$$\vec{B}_{A\square} + \vec{B}_{Ap} = 0 \Rightarrow \frac{\mu_0 I_2}{3\pi a} = \frac{2\mu_0\sqrt{2} I_1}{\pi a}$$

$$\boxed{I_2 = 6\sqrt{2} I_1 = 8{,}46\,A}$$

---

(2)



a) Φ - const, $S = A$ - const $\Rightarrow B$ - const

$$B = B_0 = 10^{-3}\,T$$

$$\Phi = B \cdot A = 25 \cdot 10^{-4} \cdot 10^{-3}\,wb = \boxed{25 \cdot 10^{-7}\,wb}$$

$$H = \frac{B}{\mu} = \frac{B_0}{\mu} = \frac{10^{-3}}{10^{-8}} \Rightarrow \boxed{10^5\,A/m}$$

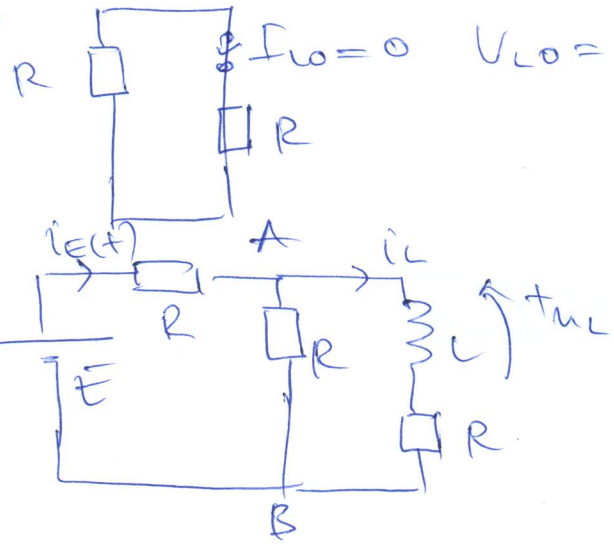
b) $\vec{F} = I\vec{\ell} \times \vec{B}_0 = I \cdot \sqrt{A} \cdot \vec{j} \times B_0 \cdot \vec{i} = I B_0 \sqrt{A}\, \vec{j} \times \vec{i}$

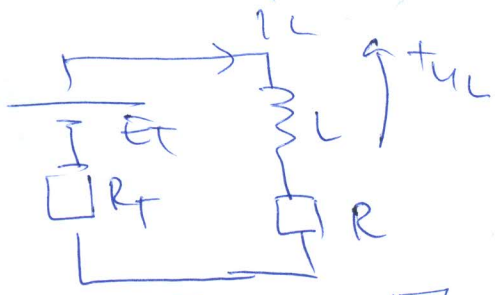
$\vec{j} \times \vec{i} = -\vec{k}$

$$\vec{F} = 10 \cdot 10^{-3} \cdot 5 \cdot 10^{-2}(-\vec{k})\ N$$

$$\boxed{\vec{F} = -5 \cdot 10^{-4}\,\vec{k}\ N}$$

(3) $n<0$





$$E_T = \frac{E}{R+R}\cdot R = E/2$$

$$R_T = R\|R = R/2$$

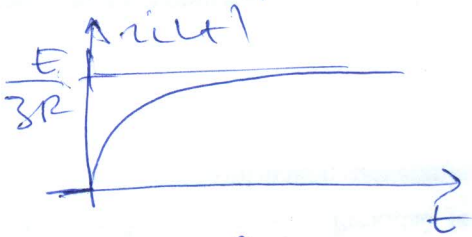
$$E_T - (R+R_T)i_L - u_L = 0$$

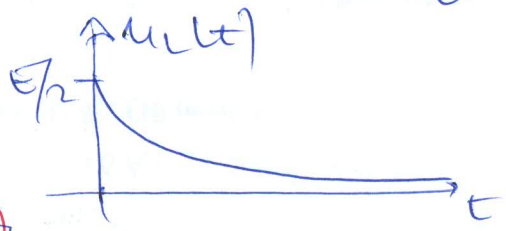
$$u_L = L\frac{di_L}{dt}$$

$$E_T - (R+R_T)i_L - L\frac{di_L}{dt} = 0$$

$$\frac{di_L}{dt} + \frac{i_L}{\frac{L}{R+R_T}} = \frac{E_T}{L} \Rightarrow$$

$$\frac{di_L}{dt} + \frac{i_L}{\frac{2L}{3R}} = \frac{E}{2L}$$





$$k = E/2L \qquad \tau = \frac{2L}{3R}$$

$$i_L(t) = Ae^{-t/\tau} + B \qquad B = k\cdot\tau = \frac{E}{3R}$$

$$A+B=0 \Rightarrow A = -B = -E/3R$$

$$i_L(t) = E/3R\,(1-e^{-t/\tau})$$

$$u_L(t) = L\frac{di_L}{dt} = \frac{LE}{3R}\frac{1}{\tau}e^{-t/\tau} = \frac{LE}{3R}\frac{3R}{2L}e^{-t/\tau} = \frac{E}{2}e^{-t/\tau}$$

b) $$u_{AB}(t) = u_L(t) + Ri_L(t) = \frac{E}{2}e^{-t/\tau} + \frac{E}{3}(1-e^{-t/\tau})$$

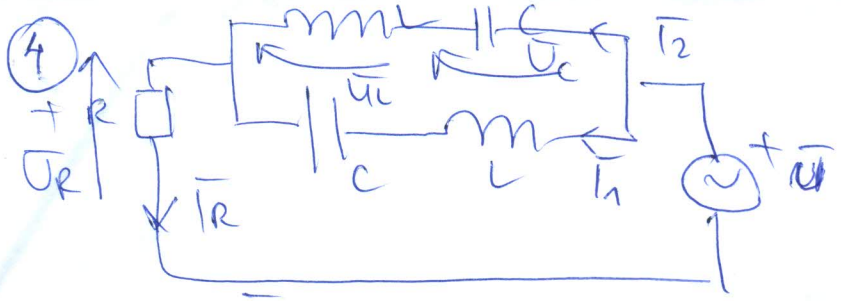
$$= E\left(\frac{1}{2}e^{-t/\tau} + \frac{1}{3} - \frac{1}{3}e^{-t/\tau}\right) = E\left(\frac{1}{6}e^{-t/\tau} + \frac{1}{3}\right) = \frac{E}{3}\left(\frac{1}{2}e^{-t/\tau} + 1\right)$$

$$u_{AB}(t) = E - Ri_E(t) \Rightarrow i_E(t) = \frac{E-u_{AB}(t)}{R}$$

$$i_E(t) = \frac{\frac{E}{2} - \frac{E}{3}\left(\frac{1}{2}e^{-t/\tau}+1\right)}{R} = \frac{E}{R}\left(1 - \frac{1}{6}e^{-t/\tau} + \frac{1}{3}\right) = \frac{E}{R}\left(\frac{2}{3} - \frac{1}{6}e^{-t/\tau}\right)$$

$$i_E\left(t=\frac{2L}{R}\right) = \frac{E}{R}\left(\frac{2}{3} - \frac{1}{6}e^{-\frac{2L/R}{2L/3R}}\right) = \frac{E}{R}\left(\frac{2}{3} - \frac{1}{6}e^{-3}\right) = \boxed{\frac{E}{6R}(4-e^{-3}) = i_E}$$

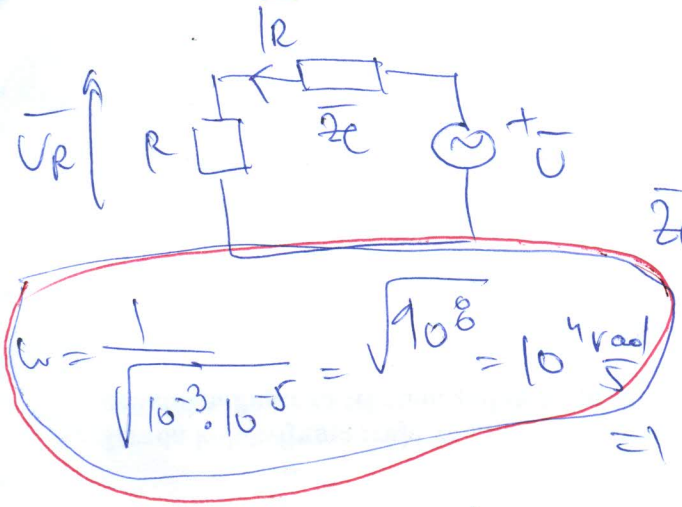
4)



$u_R(t) = u(t) \Rightarrow$ PAD NAPONA NA Ze JE ϕ !?

$\bar{U}_{Ze} = \bar{Z}_e \cdot \bar{I}_R = 0$

$\Rightarrow \bar{Z}_e = 0$



$\omega = \frac{1}{\sqrt{10^{-3} \cdot 10^{-5}}} = \sqrt{10^8} = 10^4 \frac{rad}{s}$

$\bar{Z}_e = (\bar{Z}_L + \bar{Z}_C) \| (\bar{Z}_L + \bar{Z}_C) = \frac{\bar{Z}_L + \bar{Z}_C}{2}$

$= \frac{j\omega L - j\frac{1}{\omega C}}{2} = j\,\frac{\omega L - \frac{1}{\omega C}}{2} = 0$

$\Rightarrow \omega L - \frac{1}{\omega C} = 0 \Rightarrow \boxed{\omega = \frac{1}{\sqrt{LC}}}$

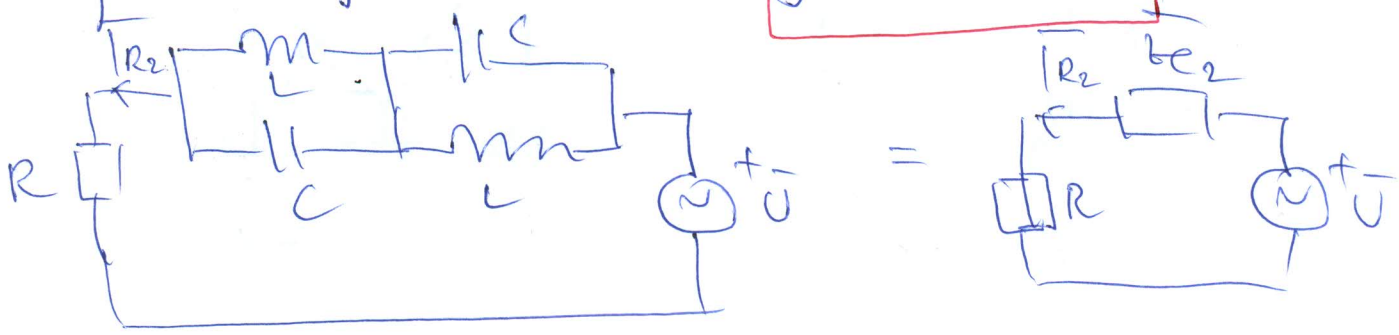
$\bar{I}_R = \frac{\bar{U}}{\bar{Z}_R} = \frac{100 e^{j0}}{10} = \boxed{10A = \bar{I}_R}$ (KOLO JE U REZONANCIJI)

$\bar{I}_1 = \bar{I}_2 = \frac{\bar{I}_R}{2} = 5A$

$\bar{U}_C = -j\frac{1}{\omega C}\bar{I}_2 = -j\frac{1}{10^4 \cdot 10^{-5}} \cdot 5 = \boxed{-j50V = \bar{U}_C}$

$\bar{U}_L = j\omega L \cdot \bar{I}_2 = j \cdot 10^4 \cdot 10^{-3} \cdot 5 = \boxed{j50V = \bar{U}_L}$

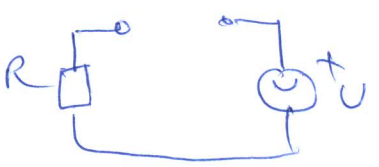
b)



$\bar{Z}_{e_2} = \bar{Z}_L \| \bar{Z}_C + \bar{Z}_L \| \bar{Z}_C = 2\,\bar{Z}_L \| \bar{Z}_C = 2 \cdot j\omega L \| \frac{1}{j\omega C}$

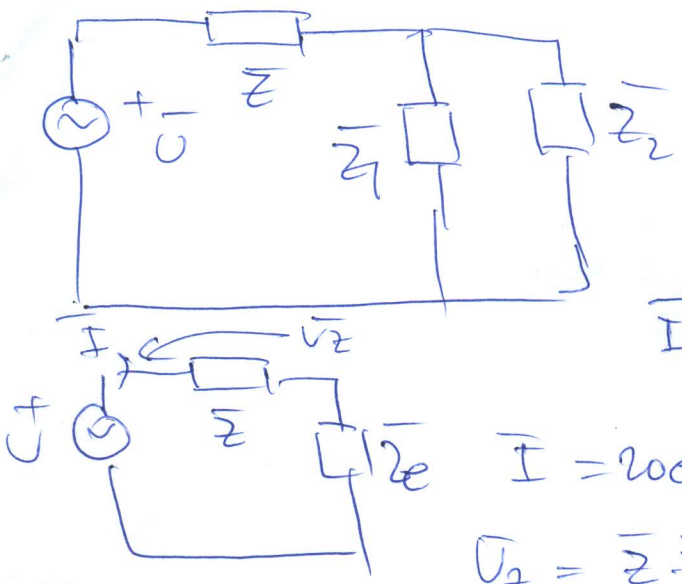
$= 2 \cdot \frac{j\omega L \cdot \frac{1}{j\omega C}}{j\omega L + \frac{1}{j\omega C}} = \frac{2L/C}{j(\omega L - \frac{1}{\omega C})} \to +\infty$ (0) $\Rightarrow Z_{e_2}$ je OTVORENA VEZA

$\Rightarrow$ NEMA STRUJE U KOLU $\Rightarrow$



$\boxed{I_{R_2} = 0A}$ (KOLO JE U ANTIREZONAN.)

(5)



a) $\bar{Z}_e = \bar{Z}_1 \| \bar{Z}_2 = \frac{\bar{Z}_1}{2} = 5e^{j\pi/6}\,\Omega$

$\bar{U} = 200e^{j0}\,V$

$\bar{I} = \frac{\bar{U}}{\bar{Z}+\bar{Z}_e} = \frac{200}{5e^{j\pi/6}+5e^{j\pi/6}} = \frac{200}{10e^{j\pi/6}}$

$\bar{I} = 20e^{-j\pi/6}\,A$

$\bar{U}_Z = \bar{Z}\cdot\bar{I} = 5e^{j\pi/6}\cdot 20e^{-j\pi/6} = 100\,V$

$U_Z = 100\,V$

$\bar{U}_{Z_2} = \bar{U}_{Z_1} = \bar{U}_{Ze} = \bar{Z}_e\bar{I} = 100\,V$

b) $\bar{S}_{Z_2} = \bar{U}_{Z_2}\cdot\bar{I}_{Z_2}^{*} = \frac{U_{Z_2}^2}{\bar{Z}_2^{*}} = \frac{10000}{10e^{-j\pi/6}} = 1000e^{j\pi/6}$

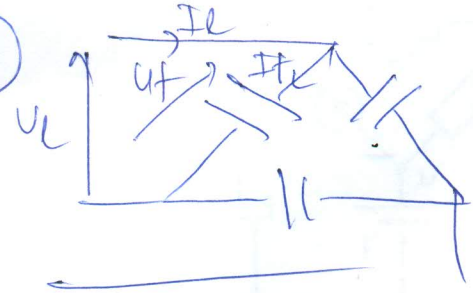
$S_{Z_2} = 1000\,VA = 1\,kVA$

$P_{Z_2} = S_{Z_2}\cos\pi/6 = 1000\cdot\frac{\sqrt{3}}{2} = 500\sqrt{3}\,W$

$Q_{Z_2} = S_{Z_2}\sin\pi/6 = 500\,VAr$

---

(6)



$U_f = U_l = 100\,V$

$Z_f = \frac{1}{\omega C} = \frac{1}{2\pi f C} = \frac{1}{2\pi\frac{50}{\pi}100\cdot10^{-6}}$

$Z_f = \frac{1}{100\cdot100\cdot10^{-6}} = \frac{1}{10^{-2}} = 100\,\Omega$

$I_f = \frac{U_f}{Z_f} = 1\,A$

$I_l = I_f\sqrt{3} = \sqrt{3}\,A$

$S = 3U_fI_f = 300\,VA$

$P = 0\,W$ (ČISTE KAPACITIVNOSTI)