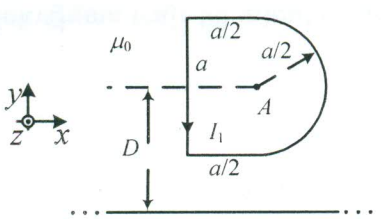
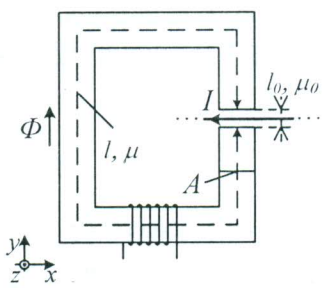
DRUGI KOLOKVIJUM IZ ELEKTROTEHNIKE
25. januar 2019.
GRUPA 2

1. Na Slici 1 prikazana je kontura koja se sastoji od kružnog luka, koji predstavlja polovinu kružnice poluprečnika $a/2$, i tri pravolinijska provodinka koji predstavljaju polovinu kvadrata stranice a=1m. Kroz konturu protiče struja intenziteta I_1=1A u označenom smeru. U istoj ravni, u vazduhu, nalazi se beskonačni pravolinijski provodnik kroz koji protiče nepoznata struja I_2. Odrediti smer i intenzitet struje I_2 tako da rezultujući vektor jačine magnetnog polja u tački **A** (centar kružnog luka) bude jednak nuli. Tačka A je udaljena D= a=1m od pravolinijskog provodnika.
(8 poena)

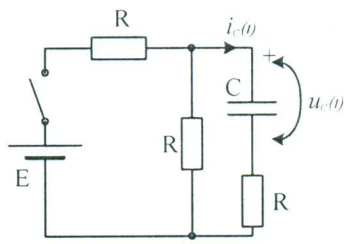
2. Na Slici 2 prikazano je magnetno kolo, koje se sastoji od jezgra, uzdužnog preseka kao na slici 2.,magnetne permeabilnosti $\mu = 2\cdot10^{-8}$ H/m i dužine srednje linije $l = 50$cm. Poprečni presek jezgra je oblika kvadrata stranice $a = 4$cm, a jezgro ima vazdušni procep debljine $l_0 = 1$mm. U procepu se nalazi pravolinijski provodnik, kroz koji protiče struja intenziteta $I = 3$A u označenom smeru. Fluks vektora magnetne indukcije u jezgru je poznat i iznosi $\Phi = 16\cdot10^{-6}$ Wb u označenom smeru.
a) Odrediti intenzitet vektora magn. indukcije u vazdušnom procepu i vektora jačine magn. polja u jezgru. **(6 poena)**
b) Odrediti i skicirati **vektor** sile kojom magnetno polje u procepu deluje na provodnik. **(6 poena)**



Slika 1



Slika 2

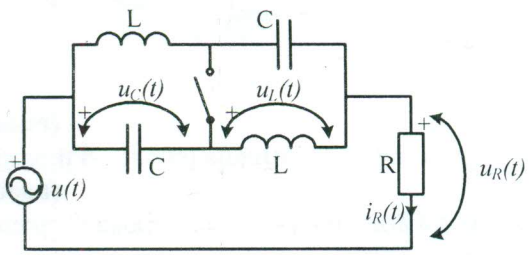


Slika 3

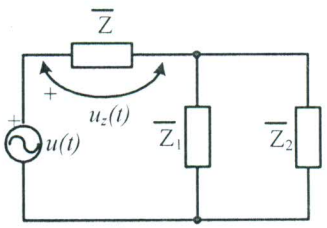
3. U kolu na Slici 3 poznate su parametri elemenata E, R i C. Prekidač П je otvoren i u kolu je uspostavljeno stacionarno stanje. U trenutku $t = 0$, prekidač se zatvara.
a) Odrediti izraze za napon i struju kondenzatora nakon zatvaranja prekidača i nacrtati odgovarajuće vremenske dijagrame. **(7 poena)**
b) Odrediti intenzitet struje generatora u trenutku t_1=3RC. **(3 poena)**

4. U kolu naizmenične struje na slici 4 poznato je: $u(t) = 100\sqrt{2}\sin(\omega t)$ V, $L = 1\mu$H, $C = 0.1$mF i $R = 10\Omega$.
a) Prekidač je otvoren, a naponi na otporniku i generatoru su isti, tj. $u_R(t) = u(t)$. Odrediti kružnu učestanost generatora ω, kompleksne oblike struje kroz otpornik $\overline{I}_R$ i napona na kondenzatoru $\overline{U}_C$ i kalemu $\overline{U}_L$. **(8 poena)**
b) Ako se prekidač zatvori, odrediti kompleksni oblik struje kroz otpornik $\overline{I}_R$. **(5 poena)**

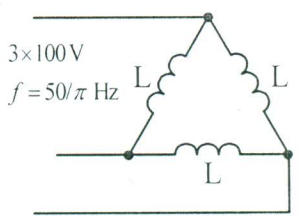
5. U kolu na slici 5 poznato je: $u(t) = 400\sqrt{2}\sin(1000t)$ V, $\overline{Z} = 10e^{j\pi/3}\Omega$, $\overline{Z}_1 = \overline{Z}_2 = 20e^{j\pi/3}\Omega$.
a) Izračunati efektivnu vrednost napona U_z na kompleksnoj impedansi $\overline{Z}$. **(4 poena)**
b) Odrediti aktivnu, reaktivnu i prividnu snagu na kompleksnoj impedansi $\overline{Z}_1$. **(6 poena)**



Slika 4



Slika 5

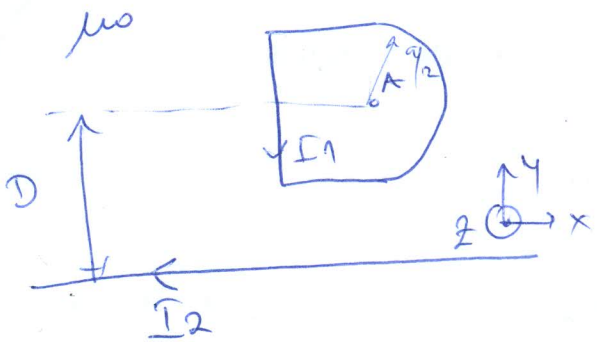


Slika 6

6. Na sistem trofaznog napona 3×100V, $f = 50/\pi$ Hz priključen je simetričan trofazni potrošač povezan u trougao, koji je sačinjen od tri kalema induktivnosti $L = 100$ mH (Slika 6). Odrediti efektivnu vrednost linijske struje, aktivnu i prividnu snagu potrošača. **(7 poena)**

G2

(1)



$$\vec{B}_{A_1} = \frac{\mu_0 I_1}{4\cdot a/2}\vec{k} + 2\cdot\frac{\mu_0 I_1}{4\pi a/2}\frac{\sqrt{2}}{2}\vec{k} + \frac{\mu_0 I_1}{4\pi a/2}\sqrt{2}\vec{k}$$

$$= \frac{\mu_0 I_1}{2a}\vec{k} + \frac{\mu_0 I_1\sqrt{2}}{2\pi a}\vec{k} + \frac{\mu_0 I_1\sqrt{2}}{2\pi a}\vec{k}$$

$$= \frac{\mu_0 I_1}{2a}\left(1+\frac{\sqrt{2}}{\pi}+\frac{\sqrt{2}}{\pi}\right)\vec{k}$$

$$= \frac{\mu_0 I_1}{2a}\left(1+\frac{2\sqrt{2}}{\pi}\right)\vec{k}$$

$$\vec{B}_{Ap} = -\frac{\mu_0 I_2}{2\pi D}\vec{k}$$

$$\vec{B}_A = \vec{B}_{A1} + \vec{B}_{Ap} = 0$$

$$\frac{\mu_0 I_1}{2a}\left(1+\frac{2\sqrt{2}}{\pi}\right) - \frac{\mu_0 I_2}{2\pi D} = 0$$

$$\frac{I_1\left(1+\frac{2\sqrt{2}}{\pi}\right)}{a} = \frac{I_2}{\pi\cdot a} \Rightarrow I_2 = I_1(\pi + 2\sqrt{2})$$

$$I_2 = 1\,A\,(3{,}14 + 2\cdot 1{,}41)$$

$$I_2 = (3{,}14 + 2{,}82)\,A = \boxed{5{,}96\,A = I_2}$$

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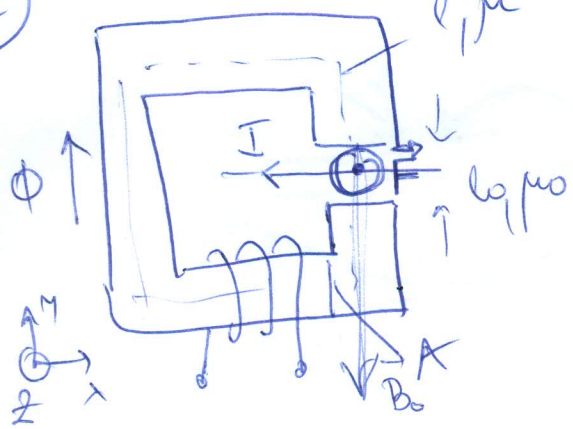
(2)



a) $\Phi = 16\cdot 10^{-6}\,wb$

$$B = \frac{\Phi}{A} = \frac{\Phi}{a^2} = \frac{16\cdot 10^{-6}\,wb}{16\cdot 10^{-4}\,m^2} = \boxed{10^{-2}\,T}$$

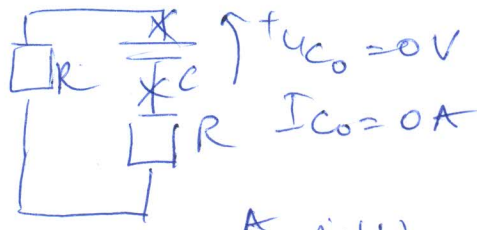
A - const, Φ - const $\Rightarrow$ $B_o - B = 10^{-2}\,T$

$$H_o = \frac{B}{\mu} = \frac{10^{-2}}{2\cdot 10^{-8}} = \boxed{0{,}5\cdot 10^6\,A/m = H}$$

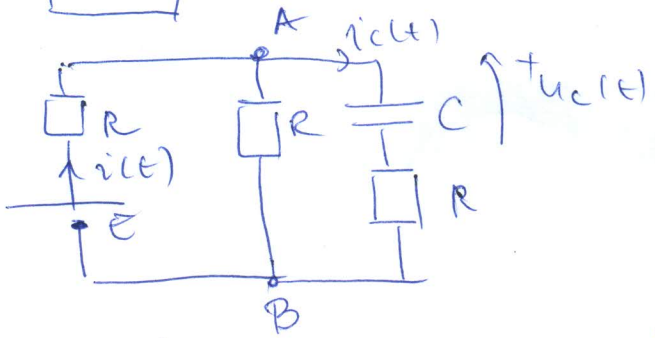
b) $\vec{F} = I\vec{l}\times\vec{B}_o = I\cdot(-\vec{i})\cdot a \times B_o\cdot(\vec{j}) = IaB_o\,\vec{i}\times\vec{j}$

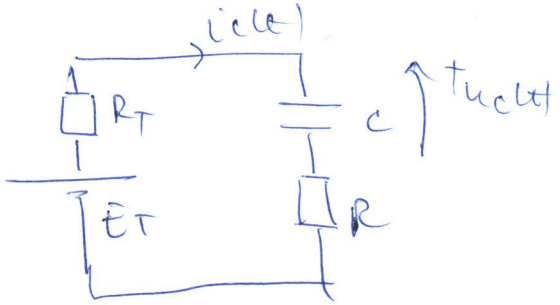
$$\vec{F} = 3{,}4\cdot 10^{-2}\cdot 10^{-2}\,\vec{k} = 12\cdot 10^{-4}\vec{k}\,N = \boxed{1{,}2\,mN\,\vec{k} = \vec{F}}$$

③ a) $t=0$



$t>0$





$$i_C = C\frac{du_C}{dt}$$

$$E_T = \frac{E}{R+R}\cdot R = E/2$$

$$R_T = R\|R = R/2$$

$$E_T - (R_T+R)\,i_C - u_C = 0$$

$$E_T - (R_T+R)C\frac{du_C}{dt} - u_C = 0$$

$$\frac{du_C}{dt} + \frac{u_C}{(R_T+R)C} = \frac{E_T}{(R_T+R)C}$$

$$\tau = C(R_T+R) = \frac{3RC}{2}$$

$$K = \frac{E_T}{(R_T+R)C} = \frac{E/2}{3RC/2} = \frac{E}{3RC}$$

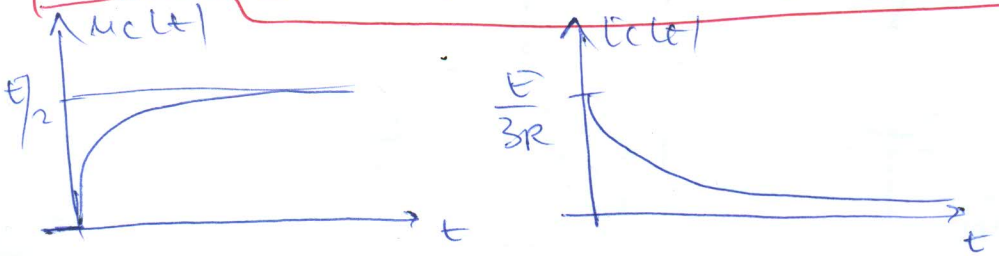
$$u_C(t) = Ae^{-t/\tau} + B$$

$$B = K\cdot\tau = E/2$$

$$A+B = U_{C_0} = 0 \Rightarrow A = -B = -E/2$$

$$\boxed{u_C(t) = E/2\,(1-e^{-t/\tau})}$$

$$i_C(t) = C\frac{du_C(t)}{dt} = \frac{CE}{2}\cdot\frac{1}{\tau}e^{-t/\tau} = \frac{CE}{2}\cdot\frac{2}{3RC}e^{-t/\tau} = \frac{E}{3R}e^{-t/\tau}$$



b) $$u_{AB}(t) = u_C(t) + Ri(t) = \frac{E}{2}(1-e^{-t/\tau}) + R\cdot\frac{E}{3R}e^{-t/\tau}$$

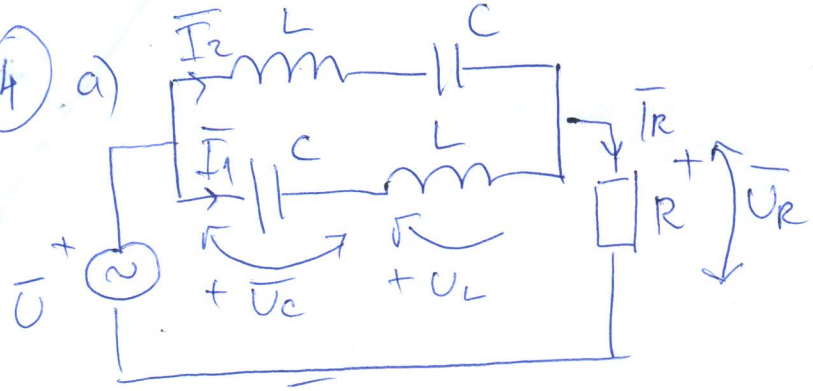
$$= E\left(\frac{1}{2} - \frac{1}{2}e^{-t/\tau} + \frac{1}{3}e^{-t/\tau}\right) = E\left(\frac{1}{2} - \frac{1}{6}e^{-t/\tau}\right)$$

$$u_{AB}(t) = \frac{E}{2}\left(1-\frac{1}{3}e^{-t/\tau}\right) = E - Ri(t) \Rightarrow i(t) = \frac{E-u_{AB}(t)}{R}$$

$$i(t) = \frac{E - \frac{E}{2} + \frac{E}{2}\cdot\frac{1}{3}e^{-t/\tau}}{R} = \frac{E/2 + E/2\cdot\frac{1}{3}e^{-t/\tau}}{R} = \frac{E}{2R}\left(1+\frac{1}{3}e^{-t/\tau}\right)$$

$$\boxed{i(t=3RC) = \frac{E}{2R}\left(1+\frac{1}{3}e^{-\frac{3RC}{3RC/2}}\right) = \frac{E}{2R}\left(1+\frac{1}{3}e^{-2}\right)}$$

(4) a)

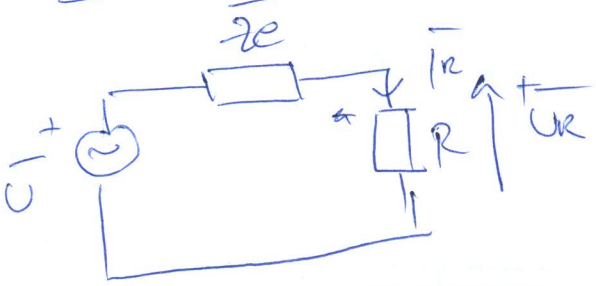


KAKO JE $u_R(t) = u(t)$

I FAZE

$\bar{U} - \bar{Z}_e \cdot \bar{I} - \bar{U}_R = 0$

$\Rightarrow \bar{Z}_e \bar{I} = 0 \Rightarrow \underline{\bar{Z}_e = 0}$



$$\bar{Z}_e = (\bar{Z}_L + \bar{Z}_C) \,||\, (\bar{Z}_L + \bar{Z}_C)$$

$$\bar{Z}_e = \frac{\bar{Z}_L + \bar{Z}_C}{2} = \frac{j\omega L - j\frac{1}{\omega C}}{2}$$

$$\bar{Z}_e = j\,\frac{\omega L - 1/\omega C}{2} = 0 \Rightarrow \boxed{\omega = \frac{1}{\sqrt{LC}}}$$

$$\omega = \frac{1}{\sqrt{10^{-6} \cdot 10^{-4}}} = 10^5 \text{ rad/s}$$

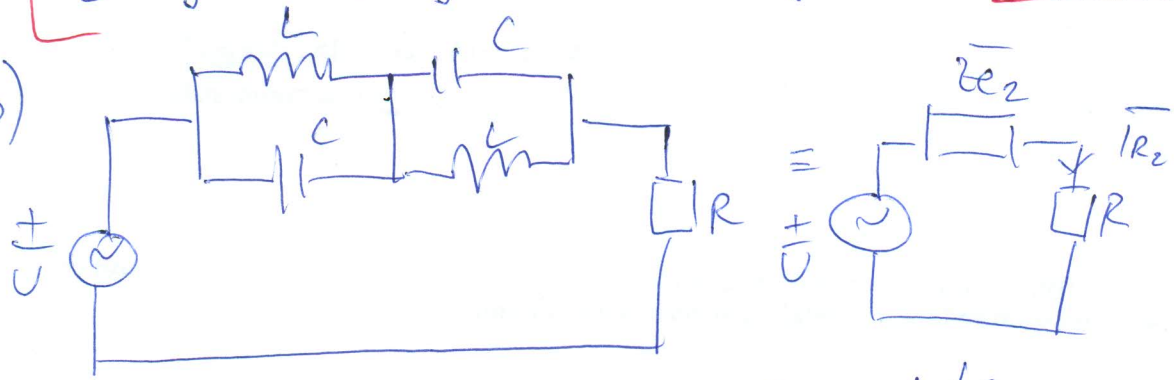
(KOLO JE U REZON.)

$$\bar{I}_R = \frac{\bar{U}}{\bar{Z}_R} = \frac{\bar{U}}{R} = \frac{100 e^{j0}}{10} = \boxed{10\text{A} = \bar{I}_R}$$

$$\bar{U}_C = -j\frac{1}{\omega C} \cdot \bar{I}_1 = -j\frac{1}{10^5 \cdot 10^{-4}} \cdot 5 = \boxed{-j95\text{V}} \qquad \bar{I}_1 = \bar{I}_2 = \frac{\bar{I}_R}{2} = 5\text{A}$$

$$\bar{U}_L = j\omega L \cdot \bar{I}_1 = j \cdot 10^5 \cdot 10^{-6} \cdot 5 = j \cdot 5 \cdot 10^{-1} = \boxed{j0{,}5\text{V}}$$

b)



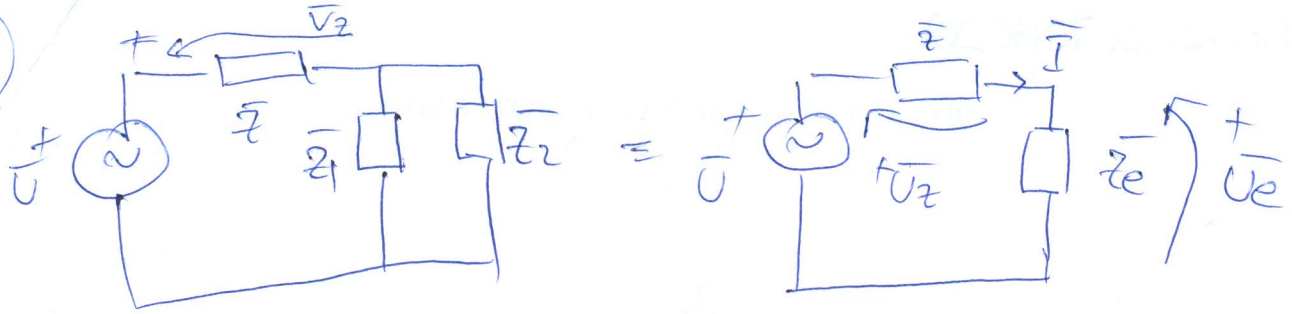
$$\bar{Z}_e = \bar{Z}_C \,||\, \bar{Z}_L + \bar{Z}_C \,||\, \bar{Z}_L = 2\,\frac{-j\frac{1}{\omega C} \cdot j\omega L}{j\omega L - j\frac{1}{\omega C}} = 2j\,\frac{L/C}{\underbrace{j\left(\omega L - \frac{1}{\omega C}\right)}_{0}} \to +\infty \quad = 1$$

$\Rightarrow \boxed{\bar{I}_R = 0}$

(KOLO JE U KNTIREZ.)

$\bar{Z}_e$ predstavlja otvorenu vezu i nema struje kroz kolo

5



a) $\bar{U} = 400\,V$

$\bar{Z}_e = \bar{Z}_1 \| \bar{Z}_2 = \frac{\bar{Z}_1}{2} = 10e^{j\pi/3}$ $\Rightarrow \bar{I} = \frac{\bar{U}}{\bar{Z} + \bar{Z}_e}$

$\bar{I} = \frac{400}{10e^{j\pi/3} + 10e^{j\pi/3}} = \frac{400}{20e^{j\pi/3}}$

$\bar{I} = 20\,e^{-j\pi/3}\,A$

$\bar{U}_Z = \bar{Z} \cdot \bar{I} = 10e^{j\pi/3} \cdot 20 \cdot e^{-j\pi/3} = 200\,V$ **$U_Z = 200\,V$**

b) $\bar{U}_e = \bar{Z}_e \cdot \bar{I} = \bar{U} - \bar{U}_Z = 200\,V$

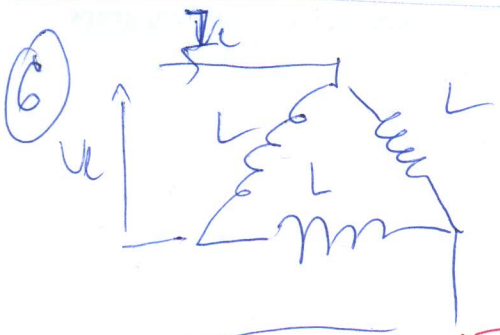
$\bar{U}_1 = \bar{U}_e = 200\,V$

$\bar{S}_1 = \bar{U}_1 \cdot \bar{I}_1^* = \frac{U_1^2}{\bar{Z}_1^*} = \frac{200^2}{20e^{-j\pi/3}} = \frac{40000}{20} e^{j\pi/3} = 2000e^{j\pi/3}$

$S_1 = 2000\,VA$ $P_1 = S_1 \cos\pi/3 = 2000 \cdot \cos\pi/3 =$ **$1000\,W = P_1$**

$Q_1 = S_1 \sin\pi/3 = 2000 \sin\pi/3 =$ **$1000\sqrt{3}\,VAr = Q_1$**

6



$U_l = 100\,V$

$U_f = U_l = 100\,V$

$I_f = \frac{U_f}{Z_f} = 10\,A$

$I_l = I_f \cdot \sqrt{3} = 10\sqrt{3}\,A$

$\bar{Z}_f = j\omega L$ $f = \frac{50}{\pi}$

$Z_f = \omega L = 2\pi f L$

$= 2\pi \cdot \frac{50}{\pi} \cdot 100 \cdot 10^{-3}$

$= 10000 \cdot 10^{-3} = 10\,\Omega$

$S = 3U_f \cdot I_f = 3 \cdot 100 \cdot 10 =$ **$3\,kVA = S$**

$P = 0\,W$ (ČISTE INDUKTIVNOSTI)