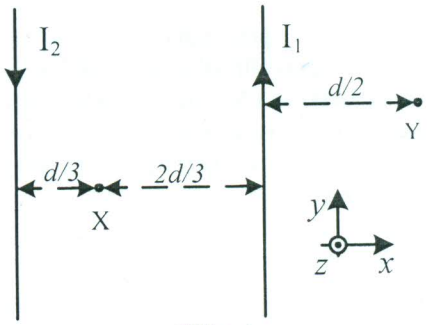
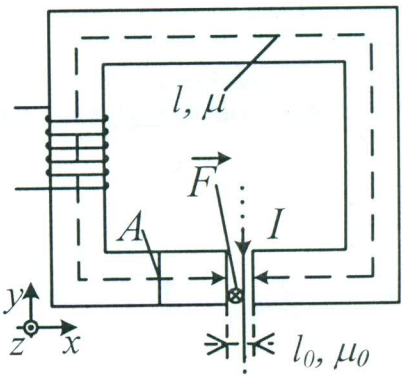
**POPRAVNI DRUGI KOLOKVIJUM IZ ELEKTROTEHNIKE**
**6. februar 2019.**
**GRUPA 1**

**1.** Na Slici 1 prikazana su dva pravolinijska, paralelna, veoma dugačka provodnika sa strujama intenziteta $I_1 = 4I$ i $I_2 = 3I$, gde je $I > 0$, koji se nalaze u vazduhu ($\mu = \mu_0$). Rastojanje između provodnika iznosi $d$.
a) Odrediti **vektor** jačine magnetnog polja u tački X. **(4 poena)**
b) Odrediti zapreminsku gustinu energije magnetnog polja u tački Y. **(6 poena)**



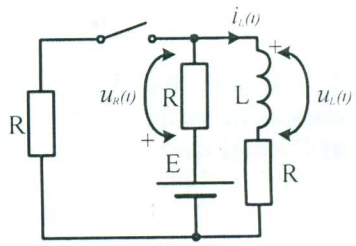
Slika 1

**2.** Na Slici 2 prikazano je magnetno kolo, koje se sastoji od jezgra, magnetne permeabilnosti $\mu = 10^{-8}\,\mathrm{H/m}$ i dužine srednje linije $l = 50\mathrm{cm}$. Poprečni presek jezgra je površine $A = 25\mathrm{cm}^2$ i oblika kvadrata, a jezgro ima vazdušni procep debljine $l_0 = 1\mathrm{mm}$. U procepu se nalazi pravolinijski provodnik, kroz koji protiče struja intenziteta $I = 10\mathrm{A}$ u označenom smeru. Vektor sile, kojom magnetno polje u procepu deluje na provodnik, iznosi $\vec{F} = 1\mathrm{mN}\cdot\left(-\vec{k}\right)$.
a) Odrediti i skicirati **vektor** magnetne indukcije u vazdušnom procepu. **(4 poena)**
b) Odrediti fluks vektora magnetne indukcije i intenzitet vektora jačine magnetnog polja unutar jezgra. **(6 poena)**



Slika 2

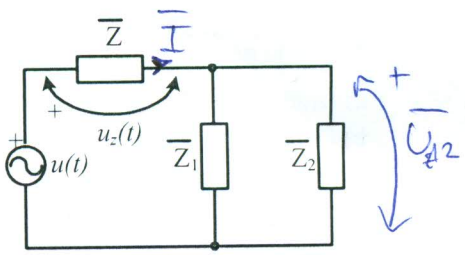
**3.** U kolu na Slici 3 poznate su parametri elemenata E, R i L. Prekidač ⊓ je zatvoren i u kolu je uspostavljeno stacionarno stanje. U trenutku $t = 0$, prekidač se otvara.
a) Odrediti izraz za struju i napon kalema nakon otvaranja prekidača i nacrtati odgovarajuće vremenske dijagrame. **(7 poena)**
b) Odrediti vrednost napona $u_R$ u trenutku $t_1$=4$L/R$. **(3 poena)**



Slika 3

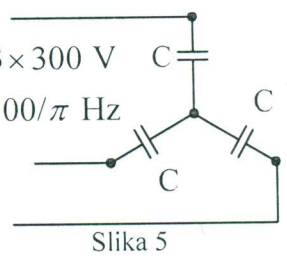
**4.** Kroz potrošač koji je priključen na izvor naizmeničnog napona $U = 500\,\mathrm{V}$, $\omega = 400\,\mathrm{rad/s}$, protiče struja efektivne vrednosti $I = 10\mathrm{A}$. Reaktivna snaga potrošača iznosi $Q = 4000\,\mathrm{var}$. Odrediti kapacitivnost kondenzatora koji treba priključiti paralelno potrošaču da bi se dobio faktor snage $\cos\varphi = 0.8$. **(10 poena)**

**5.** U kolu na Slici 4 poznato je: $u(t) = 200\sin\left(1000t + \pi/2\right)\mathrm{V}$, $\overline{Z} = 5e^{-j\pi/4}\Omega$, $\overline{Z_1} = \overline{Z_2} = 10e^{j\pi/4}\Omega$.
a) Izračunati efektivnu vrednost napona $U_z$ na kompleksnoj impedansi $\overline{Z}$. **(6 poena)**
b) Odrediti aktivnu, reaktivnu i prividnu snagu na kompleksnoj impedansi $\overline{Z}_2$. **(6 poena)**



Slika 4

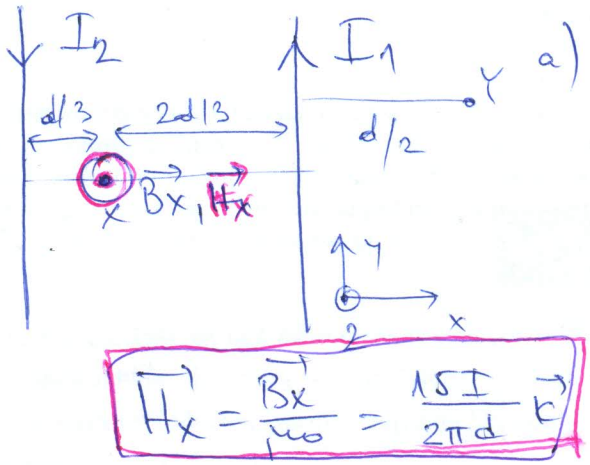
**6.** Na trofazni sistem napona $3\times300\mathrm{V}$, $f = 100/\pi$ Hz priključen je simetričan trofazni potrošač povezan u zvezdu, koji je sačinjen od tri kondenzatora kapacitivnosti $C = 50\,\mu\mathrm{F}$ (Slika 5). Odrediti efektivnu vrednost linijske struje, aktivnu i prividnu snagu potrošača. **(8 poena)**



Slika 5

G1

## 1




a) $\vec{B_x} = \frac{\mu_0 I_1}{2\pi \frac{2d}{3}}\vec{k} + \frac{\mu_0 I_2}{2\pi \cdot \frac{d}{3}}\vec{k}$

$= 3\frac{\mu_0 \cdot 4I}{4\pi d}\vec{k} + \frac{3\mu_0 3I}{2\pi d}\vec{k}$

$= \left(\frac{12\mu_0 I}{4\pi d} + \frac{29\mu_0 I}{4\pi d}\right)\vec{k}$

$= \frac{30\mu_0 I}{4\pi d}\vec{k} = \boxed{\frac{15\mu_0 I}{2\pi d}\vec{k}}$

$\boxed{\vec{H_x} = \frac{\vec{B_x}}{\mu_0} = \frac{15I}{2\pi d}\vec{k}}$

b) $\vec{B_y} = \frac{\mu_0 I_1}{2\pi \frac{d}{2}}(-\vec{k}) + \frac{\mu_0 I_2}{2\pi \cdot \frac{3d}{2}}\vec{k} = -\frac{\mu_0 \cdot 4I}{\pi d}\vec{k} + \frac{3\mu_0 I}{3\pi d}\vec{k}$

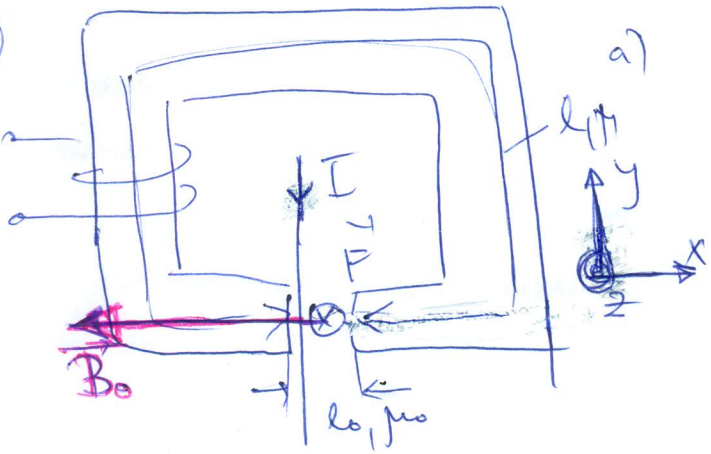
$\vec{B_y} = -\frac{3\mu_0 I}{\pi d}\vec{k}$

$\vec{H_y} = \frac{\vec{B_y}}{\mu_0} = -\frac{3I}{\pi d}\vec{k}$

$w_y = \frac{1}{2} B_y H_y = \frac{1}{2}\mu_0\left(\frac{3I}{\pi d}\right)^2$

$\boxed{w_y = \frac{9\mu_0 I^2}{2\pi^2 d^2}}$

## 2




a) $\vec{F} = I\vec{l} \times \vec{B}_0 = I\sqrt{A}\,\vec{j} \times B_0 \cdot \vec{b}_0$

$\vec{F} = I\sqrt{A}B_0\,\vec{j} \times \vec{b}_0 = -F \cdot \vec{k}$

$B_0 = \frac{F}{I\sqrt{A}} = \frac{10^{-3}\,10^{-1}}{10 \cdot 5 \cdot 10^{-2}} = \frac{10^{-2}}{5}$

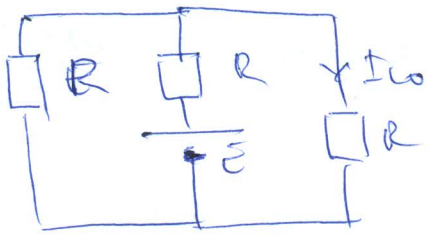
$\boxed{B_0 = 2 \cdot m T}$ $\quad \boxed{\vec{b}_0 = -\vec{i}}$

$\boxed{\vec{B}_0 = -2\,m T\,\vec{i}}$

b) $\Phi = B_0 A = 2 \cdot 10^{-3} \cdot 25 \cdot 10^{-4} = 50 \cdot 10^{-7} = \boxed{5\,\mu Wb = \Phi}$

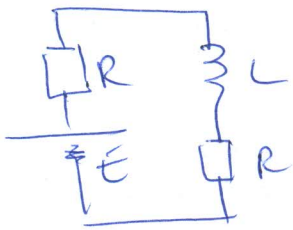
$H = \frac{B}{\mu} = \frac{B_0}{\mu} = \frac{2 \cdot 10^{-3}}{10^{-8}} = \boxed{2 \cdot 10^5\ A/m = H}$

(3) $n \to s$




$$I_{L0} = \frac{1}{2} \cdot \frac{E}{R + R/2} = \frac{E}{3R}$$

$n+0$




$$E - 2R i_L(t) - u_L(t) = 0$$

$$u_L(t) = L \frac{di_L(t)}{dt}$$

$$E - 2R i_L(t) - L \frac{di_L(t)}{dt} = 0$$

$$\frac{di_L(t)}{dt} + \frac{i_L(t)}{\frac{L}{2R}} = \frac{E}{L} \quad (K)$$

$$\frac{L}{2R} = \tau$$

$$u_L(t) = L \frac{di_L(t)}{dt}$$

$$= L \frac{-E}{6R} \cdot \frac{-1}{\tau} e^{-t/\tau}$$

$$= L \frac{E}{6R} \cdot \frac{2R}{L} e^{-t/\tau}$$

$$i_L(t) = A e^{-t/\tau} + B$$

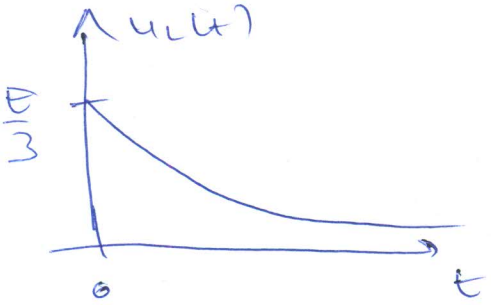
$$B = K \cdot \tau = \frac{E}{2R}$$

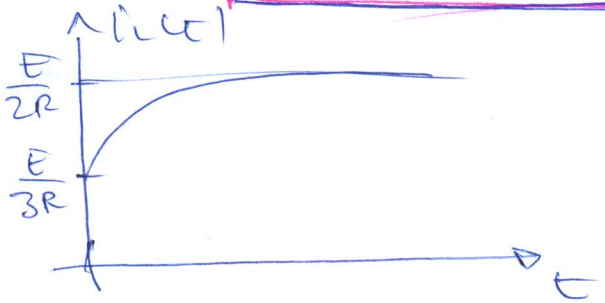
$$A + B = I_{L0} \Rightarrow A = I_{L0} - B$$

$$A = \frac{E}{3R} - \frac{E}{2R} = \frac{-E}{6R}$$

$$\boxed{u_L(t) = \frac{E}{3} e^{-t/\tau}}$$

$$\boxed{i_L(t) = -\frac{E}{6R} e^{-t/\tau} + \frac{E}{2R}}$$




b) $$i_R(t) = R \cdot i_L(t) = \frac{E}{2} - \frac{E}{6} e^{-t/\tau}$$

$$u_R\left(t_1 = \frac{4L}{R}\right) = \frac{E}{2} - \frac{E}{6} e^{-4L/R \cdot 2R/L} = \frac{E}{2}\left(1 - \frac{1}{3} e^{-8}\right)$$

$$\boxed{u_R(t_1) = \frac{E}{2}\left(1 - \frac{1}{3} e^{-8}\right) V}$$

**(4)** $Q = 4000\ VAr$, $U = 500\ V$, $\omega = 400\ rad/s$, $I = 10\ A$

PRE KONDENZATORA: $S = U \cdot I = 5000\ VA$

$P = \sqrt{S^2 - Q^2} = 3000\ W$

SA KONDENZATOROM $P' = P + P_C^{\,0} = P = 3000\ W$

$$S' = \frac{P'}{\cos\varphi} = \frac{3000}{0{,}8} = \frac{30000}{8} = \frac{15000}{4}$$

$$\sin\varphi = \sqrt{1 - \cos^2\varphi} = 0{,}6$$

$$Q' = S' \cdot \cos\varphi = \frac{1500\cancel{0}}{\cancel{4}_2} \cdot \frac{\cancel{6}3}{\cancel{10}} = \frac{4500}{2}$$

$Q' = 2250\ VAr = Q + Q_C$

$Q_C = -\omega C U^2$

$Q' = Q - \omega C U^2 \Rightarrow \boxed{C = \frac{Q - Q'}{\omega U^2}}$

$$C = \frac{4000 - 2250}{400 \cdot 250000} = \frac{1750}{100} \cdot 10^{-6} = \boxed{17{,}5\ \mu F = C}$$

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**(5)** a) $\bar{U} = \frac{200}{\sqrt{2}} e^{j\pi/2}$

$\bar{Z}_{12} = 20e^{i\pi/4} \,||\, 10e^{i\pi/4} = 5e^{i\pi/4}$

$$\bar{I} = \frac{\bar{U}}{\bar{Z} + \bar{Z}_{12}} = \frac{100\sqrt{2}e^{i\pi/2}}{5e^{-i\pi/4} + 5e^{i\pi/4}} = \frac{20\sqrt{2}e^{i\pi/2}}{2\cos\frac{\pi}{4}} = \frac{10\sqrt{2}e^{i\pi/2}}{\frac{\sqrt{2}}{2}} = 20e^{i\pi/2}$$

$\bar{U}_Z = \bar{Z} \cdot \bar{I} = 5e^{i\pi/4} \cdot 20e^{i\pi/2} = 100e^{i\pi/4}$ $\boxed{U_Z = 100\ V}$

b) $\bar{U}_{Z12} = \bar{Z}_{12} \cdot \bar{I} = 5e^{i\pi/4} \cdot 20e^{i\pi/2} = 100\,e^{i3\pi/4}$ $U_{Z12} = 100\ V$

$$\bar{S}_2 = \frac{U_{Z12}^2}{\bar{Z}_2^*} = \frac{100^2}{10e^{-i\pi/4}} = \frac{10000}{10} e^{i\pi/4} = 1000\,e^{j\pi/4}\ VA$$

$\boxed{S_2 = 1000\ VA}$ $\boxed{P = 500\sqrt{2}\ W}$ $\boxed{Q = 500\sqrt{2}\ VAr}$

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**(6)** $Z_C = \frac{1}{\omega C} = \frac{1}{2\pi f C} = \frac{1}{2\pi \cdot \frac{100}{\pi} \cdot 50 \cdot 10^{-6}} = \frac{10^6}{10000} = \frac{1000}{10} = 100\ \Omega$

$U_f = U_l/\sqrt{3} = 300/\sqrt{3}\ V$

$I_f = \frac{U_f}{Z_C} = \frac{300/\sqrt{3}}{100} = 3/\sqrt{3}\ A$

$\boxed{I_l = I_f = 3/\sqrt{3}\ A}$ $U_f = 300\ V$

$\boxed{I_l = \sqrt{3}\ A}$

$\boxed{P = 0\ W}$

$S = 3 U_f I_f = 3 \cdot 300/\sqrt{3} \cdot 3/\sqrt{3} = 900 = \boxed{900\ VA = S}$