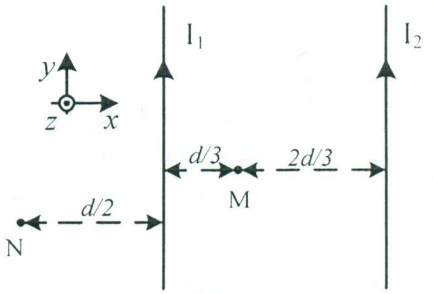
POPRAVNI DRUGI KOLOKVIJUM IZ ELEKTROTEHNIKE
6. februar 2019.
GRUPA 2

1. Na Slici 1 prikazana su dva pravolinijska, paralelna, veoma dugačka provodnika sa strujama intenziteta $I_1 = I$ i $I_2 = 6I$, gde je $I > 0$, koji se nalaze u vazduhu ($\mu = \mu_0$). Rastojanje između provodnika iznosi d.

a) Odrediti **vektor** jačine magnetnog polja u tački M. **(4 poena)**

b) Odrediti zapreminsku gustinu energije magnetnog polja u tački N. **(6 poena)**

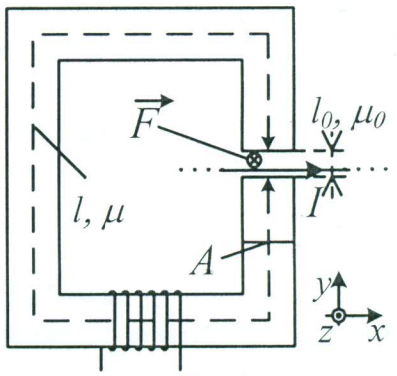


Slika 1

2. Na Slici 2 prikazano je magnetno kolo, koje se sastoji od jezgra, magnetne permeabilnosti $\mu = 2 \cdot 10^{-8}\,\mathrm{H/m}$ i dužine srednje linije $l = 50\mathrm{cm}$. Poprečni presek jezgra je oblika kvadrata stranice $a = 4\mathrm{cm}$, a jezgro ima vazdušni procep debljine $l_0 = 1\mathrm{mm}$. U procepu se nalazi pravolinijski provodnik, kroz koji protiče struja intenziteta $I = 3\mathrm{A}$ u označenom smeru. Vektor sile, kojom magnetno polje u procepu deluje na provodnik, iznosi $\vec{F} = 60\mu\mathrm{N} \cdot \left(-\vec{k}\right)$.

a) Odrediti i skicirati **vektor** magnetne indukcije u vazdušnom procepu. **(4 poena)**

b) Odrediti fluks vektora magnetne indukcije i intenzitet vektora jačine magnetnog polja unutar jezgra. **(6 poena)**

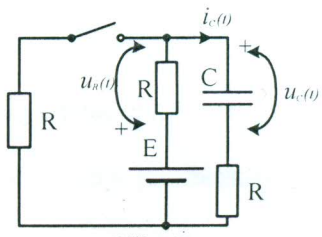


Slika 2

3. U kolu na Slici 3 poznate su parametri elemenata E, R i C. Prekidač $\sqcap$ je zatvoren i u kolu je uspostavljeno stacionarno stanje. U trenutku $t = 0$, prekidač se otvara.

a) Odrediti izraz za struju i napon kondenzatora nakon otvaranja prekidača i nacrtati odgovarajuće vremenske dijagrame. **(7 poena)**

b) Odrediti vrednost napona u_R u trenutku $t_1 = 4RC$. **(3 poena)**



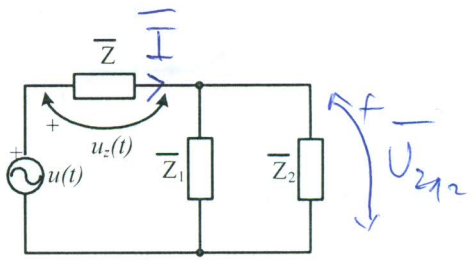
Slika 3

4. Kroz potrošač reaktanse $X = 16\,\Omega$, koji je priključen na izvor naizmeničnog napona $U = 100\,\mathrm{V}$, $\omega = 500\,\mathrm{rad/s}$, protiče struja efektivne vrednosti $I = 5\,\mathrm{A}$. Odrediti kapacitivnost kondenzatora koji treba priključiti paralelno potrošaču da bi se dobio faktor snage $\cos\varphi = 0.8$. **(10 poena)**

5. U kolu na slici 4 poznato je: $u(t) = 400\sin(1000t - \pi/2)\,\mathrm{V}$, $\overline{Z} = 10e^{j\pi/4}\,\Omega$, $\overline{Z_1} = \overline{Z_2} = 20e^{-j\pi/4}\,\Omega$.

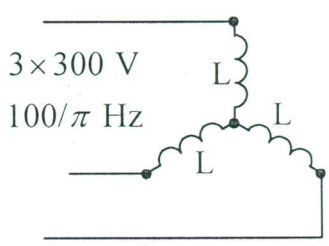
a) Izračunati efektivnu vrednost napona U_z na kompleksnoj impedansi $\overline{Z}$. **(6 poena)**

b) Odrediti aktivnu, reaktivnu i prividnu snagu na kompleksnoj impedansi $\overline{Z}_1$. **(6 poena)**



Slika 4

6. Na sistem trofaznog napona $3\times300\mathrm{V}$, $f = 100/\pi\,\mathrm{Hz}$ priključen je simetričan trofazni potrošač povezan u zvezdu, koji je sačinjen od tri kalema induktivnosti $L = 100\,\mathrm{mH}$ (Slika 5). Odrediti efektivnu vrednost linijske struje, aktivnu i prividnu snagu potrošača. **(8 poena)**



Slika 5

G2

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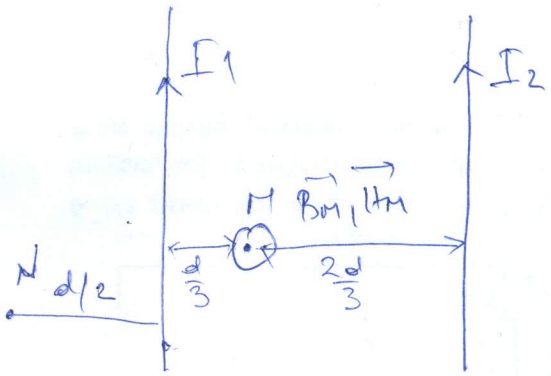

a) $\vec{B}_M = \frac{\mu_0 I_1}{2\pi \frac{d}{3}}(-\vec{k}) + \frac{\mu_0 I_2}{2\pi \cdot \frac{2d}{3}}\vec{k}$.

$\vec{B}_M = -\frac{3\mu_0 I}{2\pi d}\vec{k} + \frac{3\cdot\mu_0 6I}{4\pi d}\vec{k}$

$= -\frac{3\mu_0 I}{2\pi d}\vec{k} + \frac{9\mu_0 I}{2\pi d}\vec{k} = \frac{6\mu_0 I}{2\pi d}\vec{k}$

$\vec{B}_M = \frac{3\mu_0 I}{\pi d}\vec{k}$ $\quad \boxed{\vec{H}_M = \frac{\vec{B}_M}{\mu_0} = \frac{3I}{\pi d}\vec{k}}$

b) $\vec{B}_N = \frac{\mu_0 I_1}{2\pi d/2}\vec{k} + \frac{\mu_0 I_2}{2\pi \frac{3d}{2}}\vec{k} = \frac{\mu_0 I}{\pi d}\vec{k} + \frac{\mu_0 6I}{3\pi d}\vec{k}$

$= \frac{\mu_0 I}{\pi d}\vec{k} + \frac{2\mu_0 I}{\pi d}\vec{k} = \frac{3\mu_0 I}{\pi d}\vec{k}$

$\vec{H}_N = \frac{\vec{B}_N}{\mu_0} = \frac{3I}{\pi d}\vec{k}$ $\quad w_N = \frac{1}{2} B_N H_N = \frac{1}{2}\frac{9\mu_0 I^2}{\pi^2 d^2} = \boxed{\frac{9\mu_0 I^2}{2\pi^2 d^2} = w_N}$

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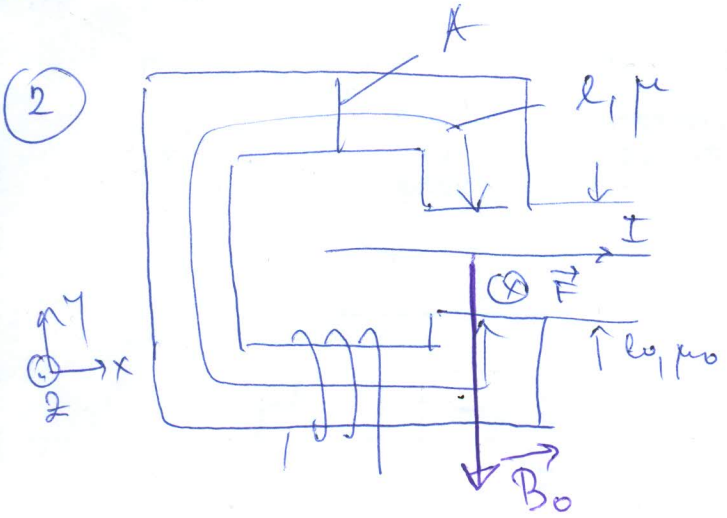


a) $\vec{F} = I\vec{\ell}\times\vec{B}_0 = I\, a\vec{i}\times B_0\vec{b}_0$

$\vec{F} = IaB_0\,\vec{i}\times\vec{b}_0 = -F\vec{k}$

$\Rightarrow B_0 = \frac{F}{Ia} = \frac{60\cdot10^{-6}}{3\cdot4\cdot10^{-2}} = 5\cdot10^{-4}\,T$

$\vec{b}_0 = -\vec{j} \Rightarrow \boxed{\vec{B}_0 = -5\cdot10^{-4}\vec{j}\ T}$

b) $\Phi = B\cdot A$ $\quad A = a^2 = 16\cdot10^{-4}\,m^2$

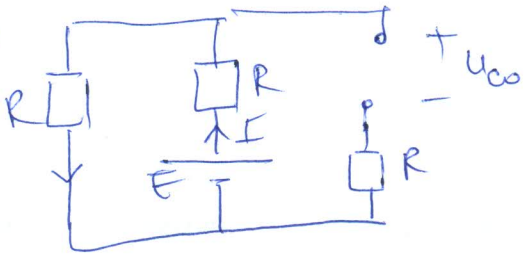
A - const, Φ - const $\Rightarrow B$ - const $\Rightarrow B = B_0 = 5\cdot10^{-4}\,T$

$\Rightarrow \boxed{\Phi = B_0\cdot a^2 = 5\cdot16\cdot10^{-8} = 8\cdot10^{-7}\,Wb}$

$H = \frac{B}{\mu} = \frac{B_0}{\mu} = \frac{5\cdot10^{-4}}{2\cdot10^{-8}} = \boxed{2{,}5\cdot10^4\ A/m = 1t}$

(3)

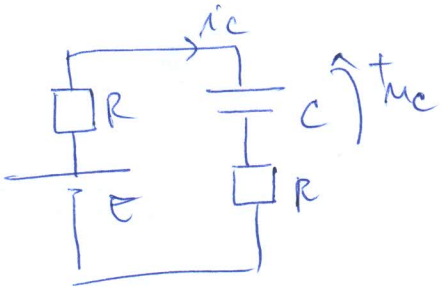
$t<0$



$$I = \frac{E}{R+R} = \frac{E}{2R}$$

$$U_{Co} = R \cdot I = \frac{E}{2}$$

$t>0$



$$E - 2R i_c(t) - u_c(t) = 0$$

$$i_c(t) = C \frac{du_c(t)}{dt}$$

$$E - 2RC \frac{du_c(t)}{dt} - u_c(t) = 0$$

$$\frac{du_c(t)}{dt} + \frac{u_c(t)}{2RC} = \frac{E}{2RC} \quad (K)$$

$$\tau = 2RC$$

$$u_c(t) = Ae^{-t/\tau} + B$$

$$i_c(t) = C \frac{du_c(t)}{dt}$$

$$B = K \cdot \tau = E$$

$$i_c(t) = \frac{C \cdot E}{2}(-1)\frac{-1}{\tau} e^{-t/\tau}$$

$$A + B = U_{Co} \Rightarrow A = U_{Co} - B$$

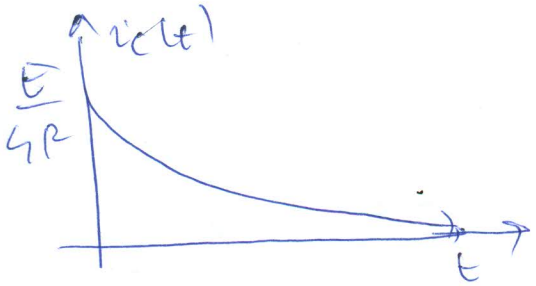
$$A = -E/2$$

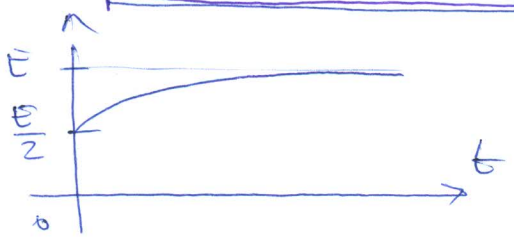
$$i_c(t) = \frac{E}{2} \frac{1}{2RC} e^{-t/\tau}$$

$$u_c(t) = -E/2\, e^{-t/\tau} + E$$

$$\boxed{i_c(t) = \frac{E}{4R} e^{-t/\tau}}$$

$$\boxed{u_c(t) = E/2\,(2 - e^{-t/\tau})}$$





b) $$u_R(t) = R\, i_c(t) = \frac{E}{4} e^{-t/\tau}$$

$$u_R(t=4RC) = \frac{E}{4} e^{-\frac{4}{\tau}} = \frac{E}{4} e^{-\frac{4RC}{2RC}} = \frac{E}{4} e^{-2}$$

$$\boxed{u_R(t_1) = \frac{E}{4} e^{-2}\ V}$$

(4) PRE VEZIVANJA KOND.

$X = 16\,\Omega$
$U = 100\,V$
$\omega = 500\,rad/s$
$I = 5\,A$

$S = UI = 500\,VA$
$Q = XI^2 = 16 \cdot 25 = 8 \cdot 50 = 400\,VAr$.
$P = \sqrt{S^2 - Q^2} = 300\,W$

KADA SE VEZE KONDENZATORI: $P' = P + P_C^{\nearrow 0} = P = 300\,W$

$Q' = Q + Q_C = Q - \omega C U^2$

$\cos\varphi = \frac{P'}{S'}$, $\sin\varphi = \frac{Q'}{S'}$ $\Rightarrow$ $\frac{P'}{Q'} = ctg\varphi = \frac{\cos\varphi}{\sin\varphi}$ $\Rightarrow$ $C = \frac{Q - Q'}{\omega U^2}$

$\sin\varphi = \sqrt{1 - 0{,}8^2} = 0{,}6$

$Q' = \frac{\sin\varphi}{\cos\varphi} \cdot P' = \frac{0{,}6}{0{,}8} \cdot 300$

$Q' = \frac{3}{4} \cdot 300 = \frac{900}{4} = \frac{450}{2} = 225\,VAr$

$C = \frac{Q - Q'}{\omega U^2} = \frac{400 - 225}{500 \cdot 100^2} = \frac{175}{5 \cdot 10^6} = 35 \cdot 10^{-6}$ $\boxed{C = 35\,\mu F}$

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(5) $\bar{U} = \frac{400}{\sqrt{2}} e^{-j\pi/2} = 200\sqrt{2}\, e^{-j\pi/2}\,V$

a) $\bar{Z}_{12} = \bar{Z}_1 \| \bar{Z}_2 = \frac{\bar{Z}_1}{2} = 10 e^{-j\pi/4}\,\Omega$

$\bar{I} = \frac{\bar{U}}{\bar{Z} + \bar{Z}_{12}} = \frac{200\sqrt{2} e^{-j\pi/2}}{10 e^{j\pi/4} + 10 e^{-j\pi/4}}$

$\bar{U}_2 = \bar{Z} \cdot \bar{I} = 10 e^{j\pi/4} \cdot 20 e^{-j\pi/2}$

$\bar{I} = \frac{20\sqrt{2} e^{-j\pi/2}}{2\cos\pi/4} = \frac{10\sqrt{2} e^{-j\pi/2}}{\frac{\sqrt{2}}{2}}$

$\bar{U}_2 = 200 e^{-j\pi/4}$ $\boxed{U_2 = 200\,V}$ $\boxed{\bar{I} = 20 e^{-j\pi/2}}$

b) $\bar{U}_{12} = \bar{Z}_{12} \cdot \bar{I} = 10 e^{-j\pi/4} \cdot 20 e^{-j\pi/2} = 200 e^{-j3\pi/4}$ $U_{12} = 200\,V$

$\bar{S}_1 = \frac{U_{12}^2}{\bar{Z}_1^*} = \frac{200^2}{20 e^{j\pi/4}} = \frac{40000}{20} e^{-j\pi/4} = 2000 e^{-j\pi/4}$

$\boxed{S_1 = 2000\,VA}$ $\boxed{P_1 = 2000 \frac{\sqrt{2}}{2} = 1000\sqrt{2}\,W}$ $\boxed{Q_1 = -1000\sqrt{2}\,VAr}$

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(6) $U_l = 300\,V$ $U_f = \frac{U_l}{\sqrt{3}} = \frac{300}{\sqrt{3}}\,V$

$\bar{Z}_L = j\omega L$ $Z_f = Z_L = \omega L = 2\pi \cdot f \cdot L = 2\pi \cdot \frac{100}{\pi} \cdot 100 \cdot 10^{-3} = 20\,\Omega$

$I_f = \frac{U_f}{Z_f} = \frac{300}{\sqrt{3} \cdot 20} = \frac{15}{\sqrt{3}}\,A$ $\boxed{I_l = I_f = \frac{15}{\sqrt{3}} = 5\sqrt{3}\,A}$

$\boxed{P = 0\,W}$

$S = 3 U_f I_f = 3 \cdot \frac{300}{\sqrt{3}} \cdot \frac{15}{\sqrt{3}} = \boxed{4500\,VA = 5}$