ПРЕДАВАЊЕ 3 – РЕШЕЊА ЗАДАТАКА ЗА ВЕЖБУ

1. $$\int \sqrt{x^2+x+1}\,dx \cdot \frac{\sqrt{x^2+x+1}}{\sqrt{x^2+x+1}} = \int \frac{x^2+x+1}{\sqrt{x^2+x+1}}\,dx =$$

$$(Ax+B)\cdot\sqrt{x^2+x+1} + \lambda\cdot\int\frac{dx}{\sqrt{x^2+x+1}} \qquad /'$$

$$\sqrt{x^2+x+1} = A\cdot\sqrt{x^2+x+1} + (Ax+B)\cdot\frac{2x+1}{2\sqrt{x^2+x+1}} + \lambda\cdot\frac{1}{\sqrt{x^2+x+1}} \qquad /2\sqrt{\ }$$

$$2(x^2+x+1) = 2A(x^2+x+1) + (Ax+B)(2x+1) + 2\lambda$$

$x^2:\ 2 = 2A+2A \qquad A = 1/2$

$x:\ 2 = 2A+A+2B \Rightarrow \qquad 2 = 3/2 + 2B \Rightarrow 2B = 1/2 \Rightarrow B = 1/4$

$x^0:\ 2 = 2A+B+2\lambda \qquad 2 = 1+\frac{1}{4}+2\lambda \Rightarrow 2\lambda = \frac{3}{4} \qquad \lambda = \frac{3}{8}$

$$\int\sqrt{x^2+x+1}\,dx = \left(\frac{x}{2}+\frac{1}{4}\right)\cdot\sqrt{x^2+x+1} + \frac{3}{8}\int\frac{dx}{\sqrt{(x+\frac{1}{2})^2+\frac{3}{4}}} =$$

$$= \left(\frac{x}{2}+\frac{1}{4}\right)\cdot\sqrt{x^2+x+1} + \frac{3}{8}\ln\left|x+\frac{1}{2}+\sqrt{x^2+x+1}\right| + C$$

2. $$\int\frac{x^3+3x}{\sqrt{-x^4-2x^2+5}}\,dx = \begin{bmatrix} t=x^2 \\ dt=2x\,dx \end{bmatrix} = \frac{1}{2}\int\frac{t+3}{\sqrt{-t^2-2t+5}}\,dt =$$

$(x^3+3x)\,dx =$

$= (x^2+3)\cdot x\cdot dx$

$$= \frac{1}{2}\left(A\cdot\sqrt{-t^2-2t+5} + \lambda\cdot\int\frac{dt}{\sqrt{-t^2-2t+5}}\right) \qquad /'$$

$$\frac{t+3}{\sqrt{-t^2-2t+5}} = A\cdot\frac{-t-1}{\sqrt{-t^2-2t+5}} + \lambda\cdot\frac{1}{\sqrt{-t^2-2t+5}} \qquad /\cdot\sqrt{-t^2-2t+5}$$

$$t+3 = A(-t-1) + \lambda \Rightarrow A = -1 \quad \lambda = 2$$

$$\int \frac{(x^3+3x)dx}{\sqrt{-x^4-2x^2+5}} = -\sqrt{-x^4-2x^2+5} + 2\int \frac{dt}{\sqrt{5-(t^2+2t+1-1)}} =$$

$$= -\sqrt{-x^4-2x^2+5} + 2\int \frac{dt}{\sqrt{6-(t+1)^2}} = -\sqrt{-x^4-2x^2+5} + \arcsin \frac{x^2+1}{\sqrt{6}} + C$$

3. $$\int \frac{\sqrt{x+1}+2}{(x+1)^2-\sqrt{x+1}}\,dx = \left[\begin{array}{l} t^2 = x+1 \\ dx = 2t\,dt \end{array}\right] = \int \frac{(t+2)\cdot 2t}{t^4-t}\,dt =$$

$$= \int \frac{2t+4}{t^3-1}\,dt = \int \frac{2t+4}{(t-1)(t^2+t+1)}\,dt =$$

$$= \int \left(\frac{A}{t-1} + \frac{Bt+C}{t^2+t+1}\right) dt = \ldots$$

4. $$\int \frac{\sqrt{x+1}+\sqrt{x-1}}{\sqrt{x+1}-\sqrt{x-1}}\,dx = \int \frac{\sqrt{\frac{x+1}{x-1}}+1}{\sqrt{\frac{x+1}{x-1}}-1}\,dx =$$

$$= \left[\begin{array}{ll} \frac{x+1}{x-1} = t^2 & x = \frac{t^2+1}{t^2-1} \\ x+1 = t^2 \cdot x - t^2 & \\ t^2+1 = x(t^2-1) & dx = \frac{2t(t^2-1)-(t^2+1)\cdot 2t}{(t^2-1)^2}\,dt = \frac{-4t\,dt}{(t^2-1)^2} \end{array}\right]$$

$$= \int \frac{t+1}{t-1} \cdot \frac{-4t\,dt}{(t^2-1)^2} = \int \frac{-4t\cancel{(t+1)}\,dt}{(t-1)^3\cdot(t+1)^{\cancel{2}}} = \int \frac{-4t\,dt}{(t+1)(t-1)^3} =$$

$$= \int \left(\frac{A}{t+1} + \frac{B}{t-1} + \frac{C}{(t-1)^2} + \frac{D}{(t-1)^3}\right) dt = \ldots$$