Perceived risk

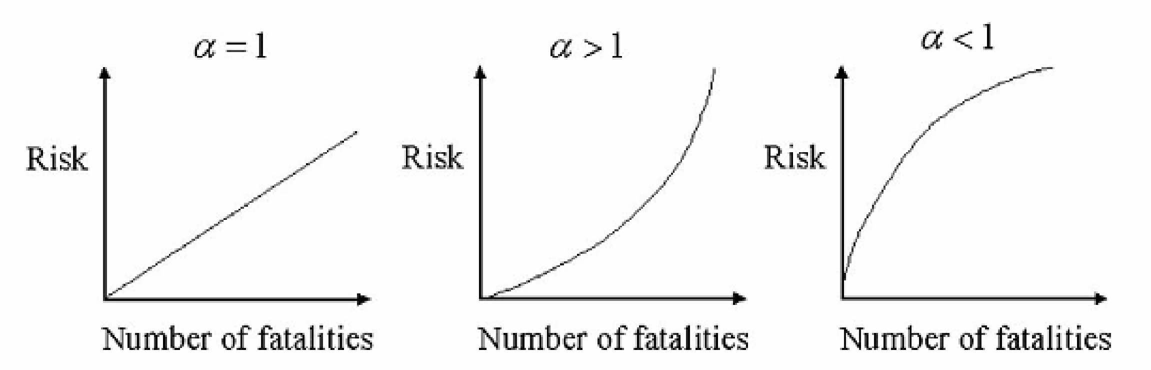
- All consequences discussed so far assume that society is risk-neutral; i.e., we are indifferent between two consequence distributions, as long as their expected values are equal.
- For example, risk neutrality assumes that a *single incident causing 100 fatalities is equivalent (or equally undesirable to the society) to 100 incidents causing one fatality each*, since in both cases the total number of fatalities is the same.
- However, most individuals would judge a low probability–high consequence (**LPHC**) event as more undesirable than a high probability–low consequence (**HPLC**) event even if the expected consequences of the two events are equal (Erkut and Verter, 1998). Consequently when dealing with **LPHC** events, most human decision makers tend to exhibit risk aversion; a single incident causing 100 fatalities is perceived as much more undesirable than 100 incidents each causing a single fatality.
- A simple way to incorporate risk attitude to risk models is to add a risk preference (or tolerance) factor ***α*** as an exponent to the consequence values. For example, if the risk assessment deals with the population exposure, then the societal risk on road segment ***l*** can be expressed as

$$R_l := s_l \iint_L p_l(D_{xy}|A,M,I)\, p_l(I|A,M)\, p_l(M|A)\, p_l(A) \times \left(POP_l(x,y)\right)^{\alpha} \,\mathrm{d}x\,\mathrm{d}y.$$

- By considering only one shipment (or one trip) and one type of hazmat spill, the traditional expected loss model of risk can thus be modified as:

$$R_l := p_l(POP_l)^{\alpha}.$$

- Figure shows three different values of α associated with three different risk preferences: ***α = 1*** models risk neutrality; ***α > 1*** models risk aversion; and ***α < 1*** models risk-taking behavior. The greater the value of ***α***, the higher the aversion to the risk of a hazmat incident. The risk-aversion model assumes that the (i + 1)st life lost is more costly than the ith life lost, for all possible values of i.

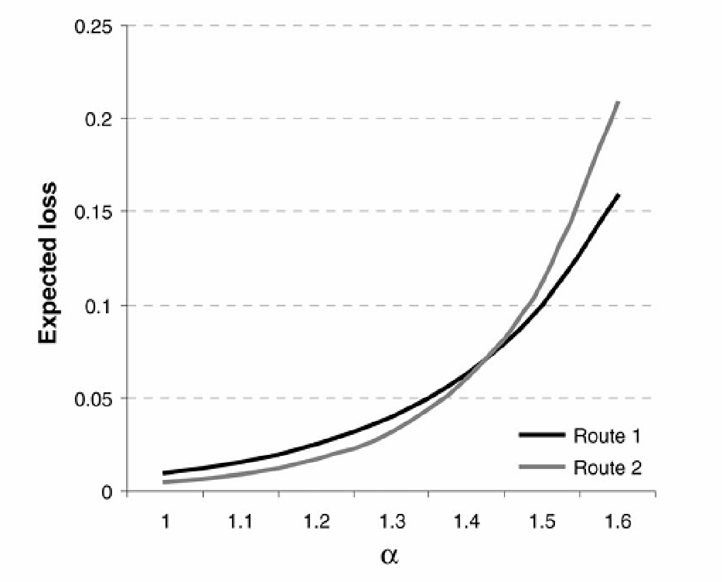


- Of course as α is increased, any route selection model that operates with an objective of minimizing total risk is eventually reduced to a model that minimizes the maximum risk, as shown by the following small example.

EXAMPLE

Consider a hazmat shipment from an origin O to a destination D. There are two routes (north and south) between O and D, P1 and P2, each consisting of two route segments. Suppose that the incident probability and the population density in the impact area of the two segments of route P1 are (10^{-4}; 25) and (10^{-4}; 75), and those of P2 are (10^{-5}; 100) and (10^{-5}; 400).

The total risks associated with P1 and P2 are 10^{-2} and 5×10^{-3}, respectively, and the maximum risks are 75×10^{-4} and 4×10^{-3}, respectively. For α = 1, we select P2, the route with lower total risk. As α approaches infinity, the problem turns into one of minimizing the maximum risk, and we select P1. Figure shows how the optimal routing decision changes from P2 to P1 as the risk-aversion factor α increases.



- The *perceived risk model can be thought of as a simple (dis)utility model*.

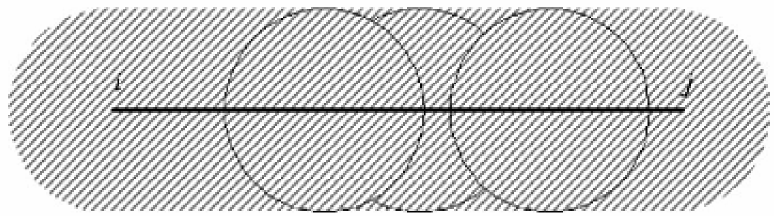
Risk on a hazmat transportation route

- Consider a road network G = (N,E) with node set N and edge set E. The nodes correspond to the origin, the destination, road intersections, and population centers and the edges correspond to road segments connecting two nodes. (We note that one does not have to model population centers as nodes if one uses a GIS)
- Note that in the context of hazmat routing it is desirable that each point on an edge has the same incident probability and level of consequence (e.g., population density). Therefore, a long stretch of a highway that goes through a series of population centers and farmland should not be represented as a single edge, but as a series of edges. Thus, a network to be used for hazmat routing is usually different from a network to be used for other transport planning purposes.

Edge risk

- Erkut and Verter (1998) proposed a risk model that takes into account the dependency to the impedances of preceding road segments
- Suppose that an edge is a collection of ***n*** unit road segments each with the same incident probability ***p*** and consequence ***c***. If, for example, the impact area of a unit road segment is modeled as a danger circle, then the impact area of an edge is a semicircular shape with the same radius as the danger circle, as shown in Figure.
- The vehicle will either have an incident on the first road segment, or it will make it safely to the second segment. If it makes it safely to the second segment, it will either have an

incident in the second segment, or it will not, and so on. They assumed that the trip ends if an incident occurs.



- Hence, the expected risk associated with this edge would be

$$pc + (1-p)pc + (1-p)^2 pc + \cdots + (1-p)^{n-1} pc.$$

- Since the incident probability p is at most on the order of 10^{-6} per trip per kilometer (based on North American data, Harwood et al., 1993), we can approximate

$$p^s \cong 0, \quad \text{for } s > 1.$$

- Consequently, the risk of hazmat transport on this edge becomes ***pnc***. For edge i, we can, thus, define the risk as

$$r_i = p_i c_i,$$

where the probability of an incident on edge i is p_i := np, and the associated consequence is c_i := c. *Note that this simple risk model works under an assumption of uniform incident probability and uniform consequence along an edge.*

- If these two attributes are not uniform, the risk computation on either an edge or an origin destination route will be more complicated.
- In practice, however, this assumption will be valid if we define long stretches of a highway as a series of edges. (In other words, it is not only the network topology, but also the value of the edge attributes that define an edge. *The edges must be short enough that the accident probability and the consequence are constant along the entire edge*.)
- This edge risk definition can be considered as a generalization of the classical (or traditional) risk definition, which considers risk as an expected loss. The expected loss can be obtained by defining n = 1, i.e., each unit road segment is considered as an edge of the road network.
- The risk of a hazmat accident on road segment l can be calculated by considering the probability that individuals in neighborhood L (of road segment l) will be affected due to the incident and the population density in L. A hazmat vehicle at point v on edge (i,j) poses a threat to the population at each point v’ in the impact area L.
- Moreover, let us assume that the consequence is determined by assuming that the impact area is a danger circle with radius λ.
- The edge-risk formulation can be derived as follows. Let l_{ij} denote the length of edge (i, j) and $w_{v'}$ denote the population density at a point v’
- The risk at point v’, $r_{v'ij}$, due to the hazmat transport on an edge (i,j) is determined by

$$r_{v',ij} := w_{v'} \int_{v=0}^{l_{ij}} \delta(v, v')\, p_{ij}(v)\, \mathrm{d}v,$$

where

$$\delta(v, v') := \begin{cases} 1, & d(v, v') \leqslant \lambda, \\ 0, & \text{otherwise}, \end{cases}$$

Hazmat routing and scheduling problems

- Routing hazmat shipments *involves a selection among the alternative paths between origin–destination pairs*. From a carrier's perspective, shipment contracts can be considered independently and a routing decision needs to be made for each shipment, which we call the ***local route planning problem***. A shipment typically involves multiple vehicles that have to be scheduled. Since the risk factors pertaining to each alternative route (such as accident probability and population exposure) can vary with time, the vehicle routing and scheduling decisions are intertwined, which we call the local routing and scheduling problem.
- *At the macro level, hazmat routing is a "many to many" routing problem with multiple origins and an even greater number of destinations*. In the sequel, we refer to this problem as ***global route planning***.
- *The **local routing problem*** is to select the route(s) between a given origin destination pair for a given hazmat, transport mode, and vehicle type. Thus, for each shipment order, this problem focuses on a single commodity and a single origin–destination route plan. Since these plans are often made without taking into consideration the big picture, certain links of the transport network tend to be overloaded with hazmat traffic.
- This could result in a considerable increase of accident probabilities on some road links as well as leading to inequity in the spatial distribution of risk. Although large-scale

hazmat carriers are known to consider transport risk in their routing and scheduling decisions, transport costs remain as the carriers' main focus.

- In contrast, the government (municipal, state/provincial, or federal) has to consider the global problem by taking into account all shipments in its jurisdiction.
- This leads to a harder class of problems that involve multi commodity and multiple origin–destination routing decisions. In addition to the total risk imposed on the public and environment, a government agency may need to consider promoting equity in the spatial distribution of risk.
- This becomes crucial in the event that certain population zones are exposed to intolerable levels of risk as a result of the carriers' routing and scheduling decisions. The governments' task is further complicated by the need to keep the transport sector economically viable – despite the regulations to ensure public safety – since dangerous goods shipments are an integral part of our industrial lifestyle.

LOCAL ROUTING PROBLEMS

- The static, deterministic and single objective local routing problems that minimize those evaluation functions reduce to the classical shortest path problem. Consequently, a label-setting algorithm (e.g., Djikstra's algorithm) can simply be applied to find an optimal route.

- Most of the exact versions of these path evaluation functions, do not satisfy all condition needed, and therefore, Djikstra's algorithm cannot be applied directly to find an optimal route.
- Kara et al. (2003) proposed a simple modification of Djikstra's algorithm to find a route that minimizes the exact version of the path incident probability. The modification relies on the adjustment of the link attribute that is used to update the node label and the scanning process.
- The algorithm is called the *impedance-adjusting node labeling shortest path algorithm* and is explained briefly as follows. Let $P = \{(i_1,i_2), (i_2,i_3,\ldots,(i_{n-1},i_n)\}$ with i_1 the origin node and i_n the destination node, and let $q(i_k)$ denote the probability of safely arriving at node i_k of P.

$q(i_k) = q(i_k-1)(1 - p_{ik-1,ik})$ for $k = 2,\ldots,n$,where $p_{ik-1,ik}$ denotes the incident probability of (i_{k-1},i_k) and **$q(i_1) := 1$**

- The attribute a_{ij} of each link (i, j) is defined as $a_{ij} := q(i)p_{ij}$. During the scanning process of node i, a_{ij} for each (i,j) is recomputed. This new value is used to update the node label θ_j of node j. If the current value of θ_j is greater than $\theta_i + a_{ij}$, then we set $\theta_j := \theta_i + a_{ij}$, $q(j) := q(i)(1 - p_{ij})$ and update the predecessor of node j.
- This modified algorithm has the same computational complexity as that of Djikstra's.
- Kara et al. (2003) also proposed the impedance-adjusting link labeling algorithm to minimize the path population exposure. This algorithm eliminates the errors resulting from double-counting of population exposure, which is caused by the network topology

- Using a similar modification technique to the impedance-adjusting node labeling shortest path algorithm, Djikstra's algorithm can be used to solve the local routing problem with the exact version of perceived risk, the expected disutility, and the mean-variance path evaluation functions.

Djikstra's algorithm

Multiobjective approaches to local routing

- Hazmat transportation is multiobjective in nature with multiple stakeholders. In general, *there is no solution that simultaneously optimizes all the conflicting objective functions in a multiobjective problem*.☞
- Instead, a set of non dominated solutions (or Pareto-optimal solutions) can be determined. ***A Pareto-optimal solution is one where we cannot improve on an objective without worsening at least one other objective.***
- Local route planning often involves finding the set of Pareto-optimal routes between a given origin destination pair. In the event that the decision maker's preferences among the conflicting objectives are available in advance, the problem can be reduced to a single objective optimization problem (via utility theory).
- Nembhard and White (1997) considered the problem of determining the most preferred path that maximizes a multi attribute, non order - preserving value function both with and without intermediate stops. For the no-stop case, the problem is solved approximately by applying the dynamic programming algorithm

- The intermediate-stop case is solved approximately by approximating the non order preserving criterion with the linear order-preserving criterion and by properly applying the dynamic programming algorithm (i.e., by using an exact method on the approximated problem).
- Marianov and ReVelle (1998) proposed a linear optimization model to find the routes that minimize both the cost and the exact version of the probability of accident. **The weighted sum technique is used to solve the bi objective problem and to approximate the set of Pareto-optimal routes**.
- The associated weighted, single objective problem can thus be solved by simply applying the classical shortest path algorithm.
- Tayi et al. (1999) dealt with the cost equity and feasibility problem in hazmat routing, where each edge of the network is associated with a vector of costs incurred by different zones due to an accident along that edge. The zones represent the community clusters, and each component of the cost vector represents the impact of an accident on a zone. The notion of cost equity is represented by six objective functions, including minimization of the average cost path, the maximum cost path, and the imbalanced cost path.
- ***However, many transport risk factors involve considerable uncertainty, which increases the difficulty of routing decisions***.
- Two methods that are frequently used in incorporating uncertainty are mean-risk and stochastic dominance.

- The mean-risk criterion is **based on comparing only two values**: the mean, representing the expected outcome; and the risk, a scalar measure of the variability of outcomes.
- **Mean–variance (MV)** criterion is probably the most well-known mean-risk criterion. It states that if E(v(P1)) <= E(v(P2)) and Var(v(P1)) <= Var(v(P2)) with at least one strict inequality, then **v(P1) is MV-strictly smaller than v(P2)**, where v(P) is an evaluation function that operates on path P.
- Stochastic dominance (SD) criterion, on the other hand, considers the entire probability distribution rather than just the two moments. It uses the cumulative distribution function (CDF) as a basis for comparison. Let FP1 and FP2 be the CDFs of two random variables v(P1) and v(P2). The first and second order stochastic dominance (FSD and SSD) are defined as follows. A random variable v(P1) is strictly smaller, with respect to FSD, than a random variable v(P2), if FP1(t) >= FP2(t) for all values of t, and at least one of the inequalities holds strictly. If two CDFs do not intersect, then one of them should stochastically dominate the other, regardless of their variances.

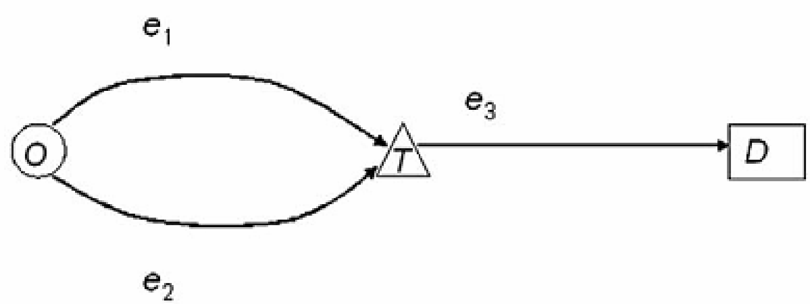
Local routing and scheduling problems

- The *traffic conditions and other risk factors in hazmat transportation networks* (e.g., incident probabilities and population exposure) *often vary with time and can at best be known a priori with uncertainty*.

- For example, for a hazmat truck, the travel time and the accident probability on certain road segments can be uncertain and depend on traffic congestion, weather conditions, and road conditions during the vehicle's trip across those links.
- Hence, the transport risk and arrival time at the destination can vary with the dispatch schedule from the origin. Also, allowing the vehicle to stop during its trip in order to avoid peak risk periods on certain road segments can be an effective strategy to reduce the total transport risk.
- To represent this phenomenon appropriately, the *transport network should be modeled as a stochastic, time-varying (STV) network*.
- In an STV network, the link attributes (such as travel times, incident probabilities, and population exposure) are represented as random variables with a priori probability distributions that vary with time. STV network-based modeling has been an important and well-researched topic since the late 1980s
- The prevailing studies can be classified into three different groups:
  - A priori optimization: the optimal routes are chosen before the travel begins. Hence, an update on the routing decision en-route is not allowed.
  - Adaptive route selection: the routing decision is subject to change en-route based on the realization of the estimated data.
  - Adaptive route selection with real-time updates: the routing decision is subject to change en-route due to real-time updates of the traffic data followed by re-optimization procedures.

A priori optimization

- ***This class of problems assumes that the optimal route is chosen before trip begins***. Hence, an update on the routing decision en-route is not allowed. All routing decisions in static (time-invariant) networks fall into this category.
- The objectives are to minimize the expected total travel time and to minimize the expected total risk as defined by the expected total number of exposed individuals.



Arcs	Travel times	Exposed populations
e_1	120 minutes (probability 1.0)	100
e_2	90 minutes (probability 0.3) 140 minutes (probability 0.7)	120
e_3	60 minutes (arrived at node *T* before 16:45) 120 minutes (otherwise)	50 (arrived at node *T* before 16:45) 200 (otherwise)

Suppose the hazmat truck must leave node O at 15:00. On the way to node T, arc e1 has both, the lowest expected travel time, and the lowest expected population exposure (120 minutes and 100 individuals as opposed to 125 minutes and 120 individuals on arc e2). Hence, Bellman's principle would include arc e1 in the optimal path.

However, note that a vehicle traveling on arc e2 has a higher probability of arriving at node T before 16:45 (0.3 probability as opposed to zero probability on arc e1).

Hence, the total expected travel time and the total expected population exposure via e2 are lower (0.3(90 + 60)+0.7(140+120) = 227 minutes and 0.3(120+50)+0.7(120+200) = 275 individuals as opposed to 240 minutes and 300 individuals).

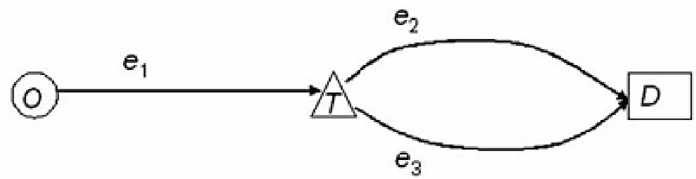
Bellman's principle

The principle that an optimal sequence of decisions in a multistage decision process problem has the property that whatever the initial state and decisions are, the remaining decisions must constitute an optimal policy with regard to the state resulting from the first decisions.

Adaptive route selection

- When traveling along a network, the motorist gathers new information that can be useful in making better routing decisions. For example, the arrival time at a node can be used in making a choice among the partial emanating from that node. This is called adaptive route selection.

- The optimal route depends on intermediate information concerning past travel times, road and weather conditions and hence, a single (and simple) path is not adequate.
- Hall (1986) showed that the optimal adaptive route in STV networks that minimizes the expected travel time is not a simple path but an acyclic subnetwork (called a hyperpath) that represents a set of routing strategies. The adaptive route specifies the road link to be chosen at each intermediate node, as a function of the arrival time at the node.



Arcs	Travel times	Exposed populations
e_1	90 minutes (0.3) 140 minutes (0.7)	100 150
e_2	60 minutes (arrived at node *T* before 16:45) 120 minutes (otherwise)	100 (arrived at node *T* before 16:45) 80 (otherwise)
e_3	100 minutes (arrived at node *T* before 16:45) 30 minutes (otherwise)	200 (arrived at node *T* before 16:45) 50 (otherwise)

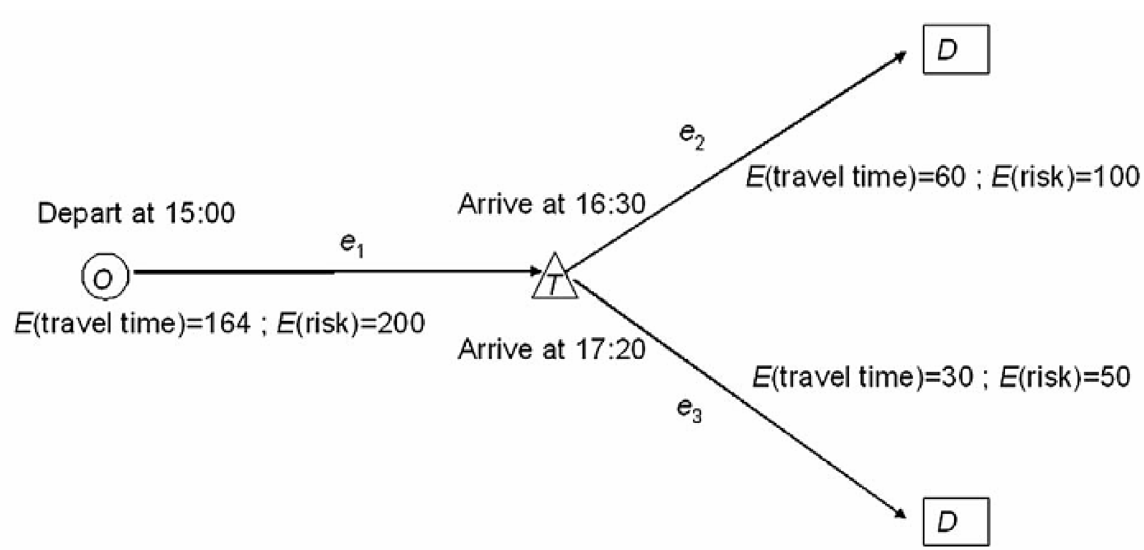
The hazmat truck is to leave node O at 15:00. The a priori expected travel times of two paths P1 := {e1,e2} and P2 := {e1,e3} are 0.3(90+60)+0.7(140+120) = 227 minutes and 0.3(90 + 100) + 0.7(140 + 30) = 176 minutes, respectively.

The associated a priori expected total risks for P1 and P2 are 0.3(100+100)+0.7(150+80) = 221 and 0.3(100+200)+0.7(150+50) = 230 individuals at risk, respectively. Hence, the a priori fastest path is path P2, and the a priori least risk path is P1.

However, if the motorist is permitted to select the rest of the path upon arrival at node T, we will obtain the following routing strategy:

- ***Travel time***: If the arrival time at node T is 16:30, take arc e2 with a travel time of 60 minutes. If the arrival time at node T is 17:20, take arc e3 with a travel time of 30 minutes. The expected travel time for the adaptive fastest path from O to D is 0.3(90+60)+0.7(140+30) = 164 minutes. The associated total risk is 0.3(100 + 100) + 0.7(150 + 50) =200 individuals at risk.
- ***Total risk***: The routing strategy is the same as for that of the adaptive fastest path.

  The resulting hyperpath of the optimal adaptive routing strategy, depicted as a decision tree, is shown in Figure



Adaptive route selection with real-time updates

- The recent advances in information and communication technologies, such as satellite-based Automatic Vehicle Location (AVL) and mobile phones, enable the driver and dispatch center to obtain and exchange real-time information. Satellite-based AVL is a computer-based vehicle tracking system that uses signals from satellite systems, such as Navstar Global Positioning System (GPS), to identify a vehicle's location. Mobile communication systems such as cellular phones, paging systems, and mobile satellite communication systems, provide two-way communication between the driver and the dispatch center or among drivers.
- These AVL and mobile communication systems enable the driver and the dispatch center to monitor and/or change the route of vehicles based on real-time information.
- These technological advances are challenging OR researchers to develop routing models and robust optimization procedures that are able to respond quickly to changes in the data. In this real-time environment, the quality of the decision depends not only on the appropriateness of the decision, but also on its timeliness (Seguin et al., 1997).
- Another main issue in this area, besides route planning, is the data updating procedure. New real-time information obtained by the dispatch center is used to update the estimation of future values of some network attributes (e.g., travel times, incident probabilities, and population in the impact area). However, this information is of limited use if the information is about parts of the transport network that are far away from the current location of the vehicle (either spatially or temporally).

- Therefore, either a spatial or a temporal discounting procedure must be applied before this real-time information is used to update the estimates of network attributes

GLOBAL ROUTING PROBLEMS

- The global route planning problem typically belongs to a government agency charged with the management of hazmat shipments within and through its jurisdiction.
- Although the transportation industry has been deregulated in many countries, hazmat transportation usually remains as part of the governments' mandate mainly due to the associated public and environmental risks.
- The two main concerns for a government agency are the total risk and the spatial distribution of risk in its jurisdiction. A number of policy tools are available to the government in mitigating public risk. These include proactive measures such as
  - the establishment of inspection stations (Gendreau et al., 2000),
  - insurance requirements (Verter and Erkut, 1997), and
  - container specifications (Barkan et al., 2000) as well as reactive measures such as
  - the establishment of hazmat emergency response networks (Berman et al., 2007).
- Another common tool for governments is to ban the use of certain road segments by potentially hazardous vehicles. For an example of such regulation, we refer the reader to the local authority bylaws section of the Alberta Dangerous Goods Transportation and Handling Act (Government of Alberta, 2002).

- In the context of global route planning, the road segments to be closed by the government can be identified by solving a hazmat network design problem.
- Equity in the spatial distribution of risk can be important for a government agency for two reasons:
  - (i) the perception of risk inequity frequently results n public opposition to the routing of vehicles carrying hazmats through the nearby passageways; and
  - (ii) the overloading of certain road segments with hazmat flows (i.e., risk inequity) may lead to an increase in the incident probabilities as well as the severity of consequences.
- The concept of equity has been studied in the OR literature primarily within the context of undesirable facility location. Marsh and Schilling (1994) provided a comprehensive review of equity measures for location problems.