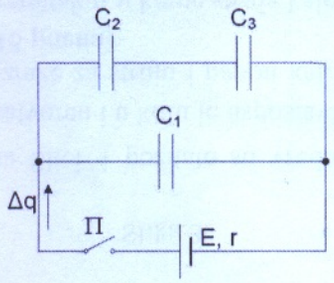
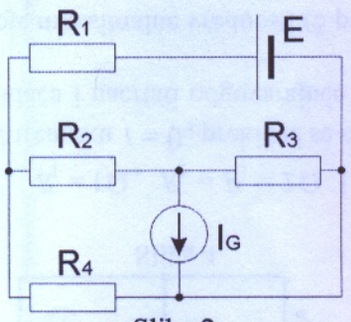
Elektrotehnika

20. februar 2023.

1. U kolu na Slici 1 poznate su kapacitivnosti kondenzatora $C_1 = 2C$, $C_2 = C_3 = C$, kao i elektromotorna sila E. Svi kondenzatori su neopterećeni, prekidač je otvoren, a u kolu je uspostavljeno stacionarno stanje.
a) Odrediti količinu naelektrisanja, Δq, koja će proteći kroz granu sa naponskim izvorom nakon zatvaranja prekidača Π. **(8 poena)**
b) Odrediti naelektrisanja, napone i elektrostatičku energiju svakog kondenzatora po uspostavljanju novog stacionarnog stanja (pri zatvorenom prekidaču). **(12 poena)**



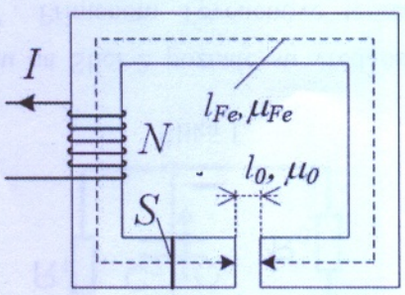
Slika 1



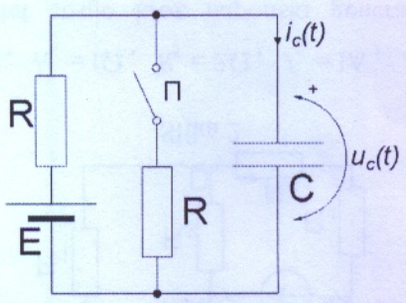
Slika 2

2. U kolu na Slici 2 poznato je: $R_1 = 10\,\Omega$, $R_2 = 5\,\Omega$, $R_3 = 5\,\Omega$, $R_4 = 20\,\Omega$, $E = 50\,\mathrm{V}$, $I_G = 20\mathrm{A}$. Odrediti struju u grani sa otpornikom R_4 primenom Tevenenove teoreme. **(20 poena)**

3. Na torus magnetne permeabilnosti μ_{Fe}, dužine srednje linije l_{Fe} i površine poprečnog preseka S, namotan je provodnik sa N navojaka, kao što je prikazano na Slici 3. Poznata je induktivnost namotaja L.
a) Odrediti debljinu vazdušnog procepa l_0. **(10 poena)**
b) Ako kroz namotaj protiče struja intenziteta I, skicirati vektor jačine magnetnog polja u vazdušnom procepu i odrediti njegov intenzitet. **(10 poena)**



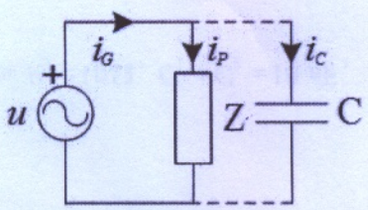
Slika 3



Slika 4

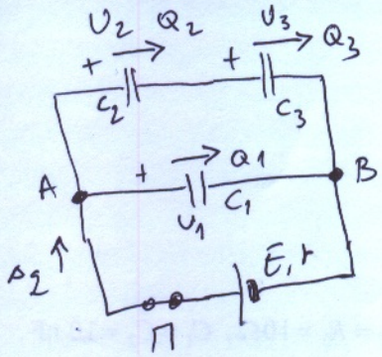
4. U kolu na Slici 4, poznate su vrednosti elemenata: E, R, C. Prekidač Π je otvoren i u kolu je uspostavljeno stacionarno stanje. U trenutku $t = 0$, prekidač se zatvara.
a) Odrediti izraze za napon i struju kondenzatora nakon zatvaranja prekidača i nacrtati odgovarajuće vremenske dijagrame. **(15 poena)**
b) Odrediti trenutak t_1 u kome napon kondenzatora dostiže 75% svoje maksimalne vrednosti. **(5 poena)**

5. U kolu naizmenične struje (Slika 5), potrošač kompleksne impedanse $\overline{Z} = (1 + j)\,\Omega$ priključen je na naponski izvor $u(t) = 100 \cdot \sin(100 \cdot t)\,\mathrm{V}$.
a) Odrediti kapacitivnost kondenzatora koji je potrebno priključiti paralelno potrošaču da bi se faktor snage popravio na vrednost $\cos\varphi = 1$. **(10 poena)**
b) Nacrtati fazorski dijagram za sve električne veličine označene na slici, u situaciji nakon priključenja kondenzatora. **(10 poena)**



Slika 5

1



a) $C_{23} = \frac{C_2 C_3}{C_2 + C_3} = \frac{C \cdot C}{2C} = C/2$

$C_1 = 2C$

$C_{AB} = C_{123} = C_1 + C_{23} = 2C + \frac{1}{2}C = \frac{5}{2}C$

$U_{AB} = E$

$\Delta Q = Q_{AB} = U_{AB} \cdot C_{AB} = \frac{5}{2} CE$

b) $Q_1 = C_1 \cdot U_1 = 2CE$

$U_1 = U_{AB} = E$

$W_1 = \frac{1}{2} Q_1 U_1 = 2CE^2$

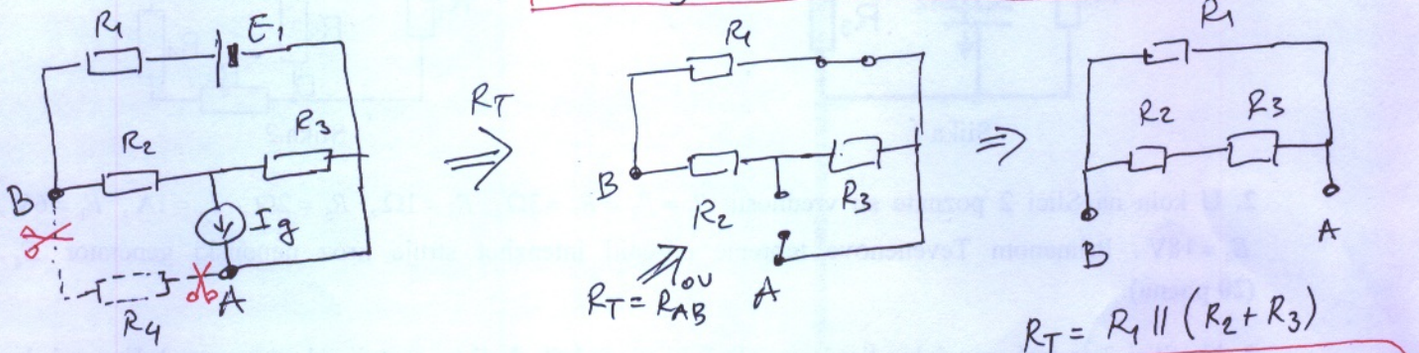
$Q_2 = Q_3 = Q_{23} = C_{23} \cdot U_{AB} = \frac{C}{2} \cdot E$

$Q_2 = Q_3 = \frac{1}{2} CE$

$U_2 = \frac{Q_2}{C_2} = \frac{1}{2} E$ $\quad W_2 = \frac{1}{2} Q_2 U_2 = \frac{1}{8} CE^2$

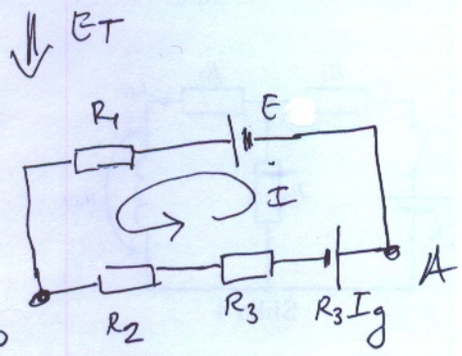
$U_3 = \frac{Q_3}{C_3} = \frac{1}{2} E$ $\quad W_3 = \frac{1}{2} Q_3 U_3 = \frac{1}{8} CE^2$

2



$R_T = R_1 \,||\, (R_2 + R_3)$

$R_T = \frac{R_1 \cdot (R_2 + R_3)}{R_1 + R_2 + R_3} = 5\Omega$

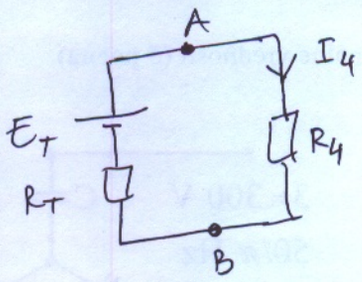


$E_T = U_{AB}^{ov}$

$I = \frac{E + R_3 I_g}{R_1 + R_2 + R_3} = \frac{50 + 100}{20} = 7{,}5\,A$

$U_{AB}^{ov} = R_1 I - E = 75 - 50 = 25V$

$E_T = U_{AB}^{ov} = 25V$



$I_4 = \frac{E_T}{R_T + R_4} = \frac{25}{20 + 5} = 1A$

3 a) $$L = \frac{N^2}{R_{m0} + R_{mFe}} = \frac{N^2}{\frac{l_0}{\mu_0 S} + \frac{l_{Fe}}{\mu_{Fe} S}}$$

$$\frac{l_0}{\mu_0 S} + \frac{l_{Fe}}{\mu_{Fe} S} = \frac{N^2}{L}$$

$$\frac{l_0}{\mu_0} = \frac{N^2 S}{L} - \frac{l_{Fe}}{\mu_{Fe}}$$

$$\boxed{l_0 = \frac{\mu_0 N^2 S}{L} - \frac{\mu_0 l_{Fe}}{\mu_{Fe}}}$$

ALTERNATIVO:

b) $$\phi = \frac{\Psi}{N} = \frac{LI}{N}$$

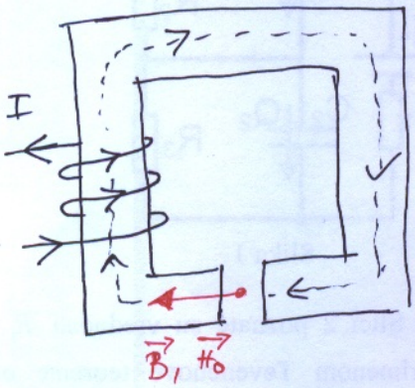
$$B = \frac{\phi}{S} = \frac{LI}{NS}$$

$$H_0 = \frac{B}{\mu_0} = \frac{LI}{\mu_0 NS}$$

b) $$H_0 l_0 + H l_{Fe} = NI$$

$$\frac{B l_0}{\mu_0} + \frac{B l_{Fe}}{\mu_{Fe}} = NI$$

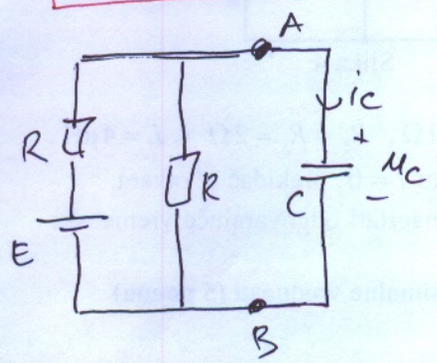
$$B = \frac{NI}{\frac{l_0}{\mu_0} + \frac{l_{Fe}}{\mu_{Fe}}}$$



$$H_0 = \frac{B}{\mu_0} = \frac{NI}{l_0 + \frac{\mu_0}{\mu_{Fe}} l}$$

$$H_0 = \frac{NI}{\frac{\mu_0 N^2 S}{L} - \frac{\mu_0 l_{Fe}}{\mu_{Fe}} + \frac{\mu_0 l_{Fe}}{\mu_{Fe}}} = \boxed{\frac{I L}{\mu_0 N S} = H_0}$$

4 $$\boxed{u_{c0} = u_c(0) = E}$$



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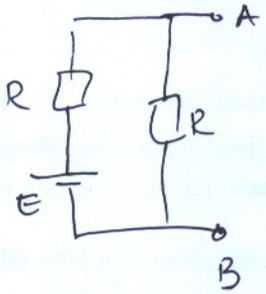
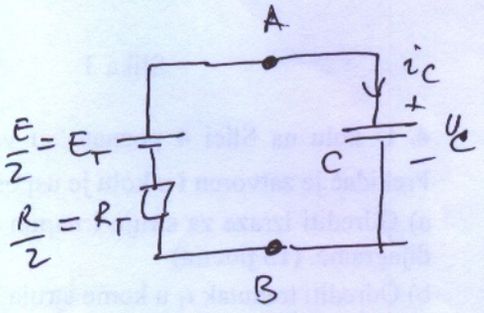




$$E_T = U_{AB}^{OY} = \frac{E}{2}$$

$$R_T = R_{AB}^{OY} = R \parallel R = R/2$$

$$E_T - R_T i_C - u_C = 0$$

$$\frac{R}{2} i_C + u_C = \frac{E}{2}$$

$$\frac{Rc}{2} \frac{du_C}{dt} + u_C = E/2 \;\Big/\; \frac{2}{Rc}$$

$$\boxed{\frac{du_C}{dt} + \frac{u_C}{\left(\frac{Rc}{2}\right)} = \frac{E}{Rc}}$$

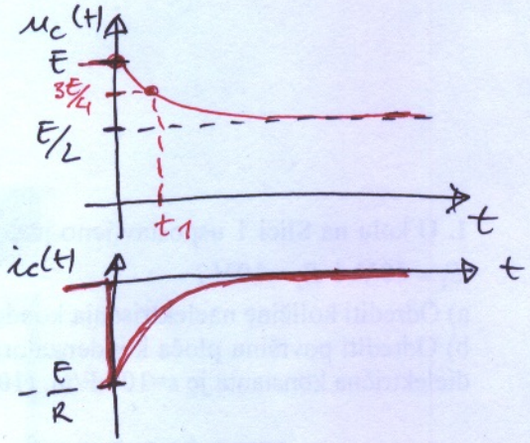
τ k

$u_c(t) = A e^{-\frac{t}{\tau}} + B$, $B = kT = \frac{E}{2}$, $U_{c0} = A + B \Rightarrow A = U_{c0} - B = E - \frac{E}{2} = \frac{E}{2}$

$$u_c(t) = \frac{E}{2} e^{-\frac{t}{RC/2}} + \frac{E}{2}$$

$$i_c(t) = C \frac{du_c}{dt} = C \cdot \frac{E}{2} \cdot \left(-\frac{2}{RC}\right) \cdot e^{-t/(RC/2)}$$

$$i_c(t) = -\frac{E}{R} e^{-\frac{t}{(RC/2)}}$$



b) $U_{cmax} = 75\% \cdot E = \frac{3}{4} E$

$$u_c(t_1) = \frac{E}{2} e^{-\frac{t_1}{RC/2}} + \frac{E}{2} = \frac{3}{4} E$$

$$\frac{E}{2} e^{-\frac{t_1}{RC/2}} = \frac{3}{4}E - \frac{1}{2}E = \frac{1}{4}E \quad / \cdot \left(\frac{2}{E}\right)$$

$$e^{-\frac{t_1}{RC/2}} = 1/2 \quad / \ln(\cdot)$$

$$-t_1 = \frac{RC}{2} \ln\left(\frac{1}{2}\right)$$

$$t_1 = \frac{RC}{2} \ln(2)$$

ALTERNATIVNO:

a) $\bar{Y} = \frac{1}{2} = \frac{1}{1+j} \cdot \frac{1-j}{1-j} = \frac{1}{2} - j\frac{1}{2}$ S

$\bar{Y}_C = j\omega C$

$\bar{Y}' = \bar{Y} + \bar{Y}_C = \frac{1}{2} + j\left(\omega C - \frac{1}{2}\right)$ S

$\cos\varphi = 1 \Rightarrow \gamma = 0 \Rightarrow \omega C = \frac{1}{2}$

$C = \frac{1}{2\omega} = 5$ mF

5

a) $U_m = 100$ V $\Rightarrow U = \frac{U_m}{\sqrt{2}} = \frac{100}{\sqrt{2}} = 50\sqrt{2}$ V

$\bar{Z} = (1+j)\,\Omega$

$$\bar{S} = \frac{U^2}{\bar{Z}^*} = \frac{(50\sqrt{2})^2}{1-j} \cdot \frac{1+j}{1+j} = \frac{2500 \cdot 2}{2}(1+j) = \underbrace{2500}_{P} + j\underbrace{2500}_{Q}\ \text{VA}$$

$P = 2500$ W, $Q = 2500$ VAR

$\bar{S}' = P' + jQ' \Rightarrow \cos\varphi = \frac{P'}{S'} = 1 \Rightarrow S' = P'$, $Q' = 0$

$P' = P + \cancel{P_C}^{\,0} = P = 2500$ W

$0 = Q' = Q + Q_C \Rightarrow Q_C = -Q = -2500$ VAR

$Q_C = -\omega C U^2 \Rightarrow$ $$C = \frac{-Q_C}{\omega U^2} = \frac{2500}{100 \cdot (50\sqrt{2})^2} = \frac{2500}{100 \cdot 2500 \cdot 2} = 5\ \mu\text{F}$$

b) $\bar{U} = U = 50\sqrt{2}$ V

$\bar{I}_p = \frac{\bar{U}}{\bar{Z}} = \frac{50\sqrt{2}}{1+j} \cdot \frac{1-j}{1-j} = 25\sqrt{2}(1-j)\,\text{A} = \bar{I}_p$

$\bar{Z}_C = -j\frac{1}{\omega C} = -j2\,\Omega \Rightarrow \bar{I}_C = \frac{\bar{U}}{\bar{Z}_C} = \frac{50\sqrt{2}}{-j2} = j25\sqrt{2}\,\text{A} = \bar{I}_C$

$\bar{I}_G = \bar{I}_p + \bar{I}_C = 25\sqrt{2}$ A