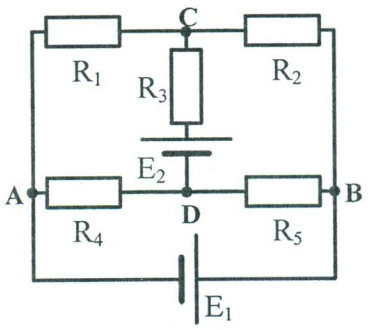
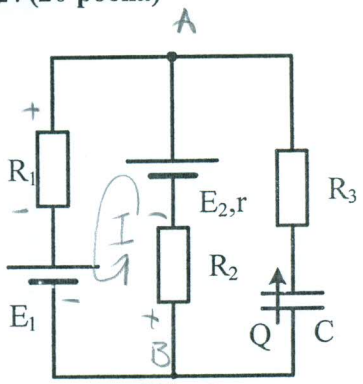
Elektrotehnika

15. septembar 2022.

1. U kolu na Slici 1, primenom Tevenenove teoreme odrediti struju generatora E_2. Poznato je $E_1 = E_2 = 12\text{V}$, $R_1 = 1\Omega$, $R_2 = 2\Omega$, $R_3 = 2\Omega$, $R_4 = 5\Omega$, $R_5 = 10\Omega$. **(20 poena)**



Slika 1



Slika 2

2. U kolu na Slici 2 poznato je: $E_1 = 15\text{V}$, $E_2 = 21\text{V}$, $r = 6\Omega$, $R_1 = R_2 = 12\Omega$ i $C = 5\mu\text{F}$.

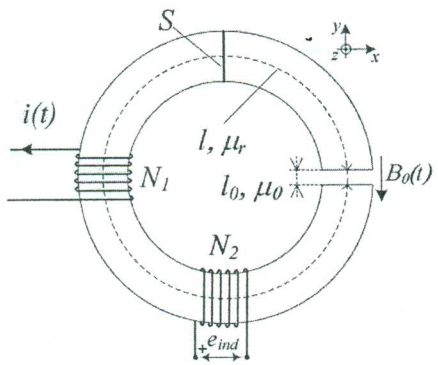
a) Odrediti količinu naelektrisanja kondenzatora. **(10 poena)**

b) Ako je kondenzator C pločasti, odrediti intenzitet vektora električnog polja između ploča, ako je rastojanje između njih $D = 0.1\text{mm}$. **(5 poena)**

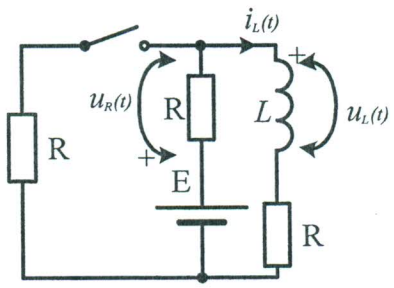
3. Na Slici 3 prikazano je magnetno kolo, koje se sastoji od jezgra, relativne magnetne permeabilnosti μ_r, dužine srednje linije l, sa vazdušnim procepom debljine l_0. Poprečni presek jezgra iznosi S. Namotaj sa N_2 navojaka je otvorenih krajeva, a kroz namotaj sa N_1 navojaka protiče struja nepoznatog intenziteta. Poznato je da se u vazdušnom procepu intenzitet vektora magnetne indukcije $\vec{B}_0(t)$ menja u vremenu po formuli $B_0(t) = B_{0m}\cos\omega t$, gde su B_{0m} i ω poznate konstante.

a) Odrediti trenutnu vrednost struje $i(t)$ kroz namotaj sa N_1 navojaka. **(10 poena)**

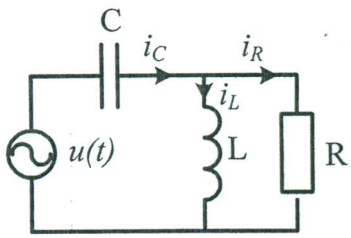
b) Odrediti efektivnu vrednost indukovane ems (E_{ind}) u namotaju čiji su krajevi otvoreni. **(10 poena)**



Slika 3



Slika 4



Slika 5

4. U kolu na Slici 4 poznati su parametri elemenata E, R i L. Prekidač je otvoren i u kolu je uspostavljeno stacionarno stanje. U trenutku $t = 0$, prekidač se zatvara.

a) Odrediti izraz za struju i napon kalema nakon zatvaranja prekidača i nacrtati odgovarajuće vremenske dijagrame. **(15 poena)**

b) Odrediti vrednost napona u_R u trenutku $t_1 = 2R/3L$. **(5 poena)** $t_1 = 2L/3R$

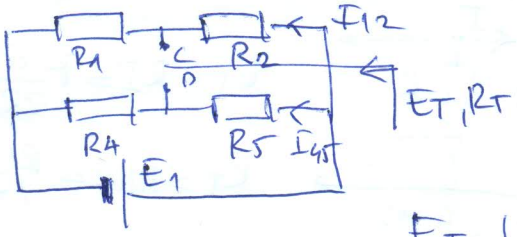
5. Potrošač, koga čine elementi $C = 2\text{mF}$, $L = 50\text{mH}$ i $R = 5\Omega$, napaja se iz izvora naizmeničnog napona $u(t) = 20\sin(\omega t)\text{V}$, gde je $\omega = 100\text{rad/s}$ (Slika 5).

a) Odrediti kompleksne izraze za označene struje **(10 poena)**

b) Odrediti trenutnu vrednost struje kalema. **(5 poena)**

c) Odrediti aktivnu, reaktivnu i prividnu snagu celokupnog potrošača. **(10 poena)**

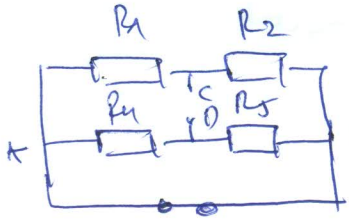
(1)



$$I_{12} = \frac{E_1}{R_1+R_2} = 4A$$

$$I_{45} = \frac{E_1}{R_4+R_5} = 0{,}8A$$

$$E_T = U_{CD}^{ov} = R_1 I_{12} - R_4 I_{45} = \boxed{0V = E_T}$$



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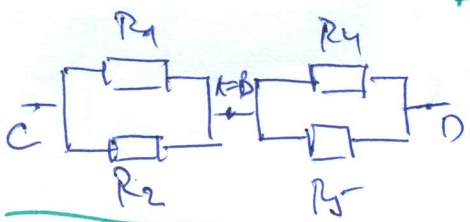
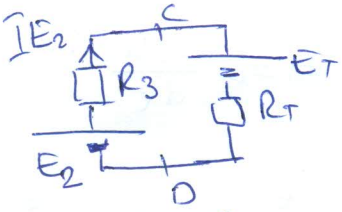


$$R_T = R_1 \| R_2 + R_4 \| R_5$$

$$= 4\Omega = R_T$$



$$I_{E2} = \frac{E_2 - E_T}{R_2 + R_T} = 2A$$

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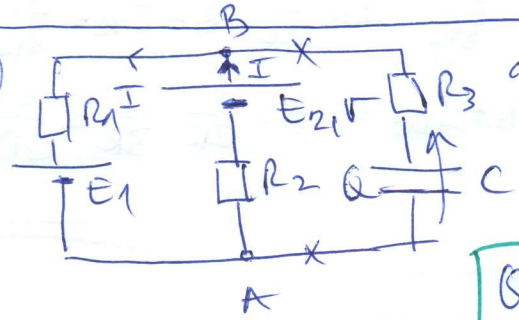
(2)



a) $U_C = U_{AB} = -E_1 - R_1 I = \boxed{-\frac{87}{5}V = U_C}$

$$I = \frac{E_2 - E_1}{R_2 + r + R_1} = \frac{1}{5}A$$

$$\boxed{Q_C = U_C \cdot C = -87\mu C}$$

b) $K = \frac{|U_C|}{D} = +\frac{87}{5 \cdot 10^{-4}} = \boxed{+174\frac{kV}{m} = K}$

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(3)

a) $\oint_C \vec{H}(t)\,d\vec{l} = \Sigma i(t)$

$H(t)\,l + H_0(t)\,l_0 = N_1 i(t)$

$\frac{B(t)\,l}{\mu_0 \mu_r} + \frac{B(t)\,l_0}{\mu_0} = N_1 i(t)$

$$\Rightarrow i(t) = \frac{B(t)}{\mu_0 N_1}\left(\frac{l}{\mu_r} + l_0\right)$$

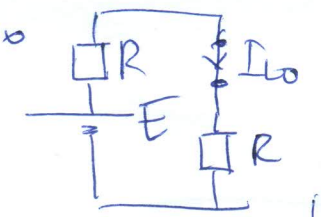
$$\boxed{i(t) = \frac{B_0}{\mu_0 N_1}\left(\frac{l}{\mu_r} + l_0\right)\cos\omega t}$$

$E_{IND\,MAX}$

b) $e_{ind} = -N_2 \frac{d\Phi(t)}{dt} = -N_2 S \frac{dB(t)}{dt} = +N_2 S B_0 \omega \sin\omega t$

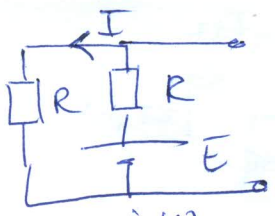
$$\boxed{E_{ind} = \frac{E_{IND\,MAX}}{\sqrt{2}} = \frac{N_2 S B_0 \omega}{\sqrt{2}}}$$

④ a) $t<0$



$I_{L0} = \frac{E}{2R}$

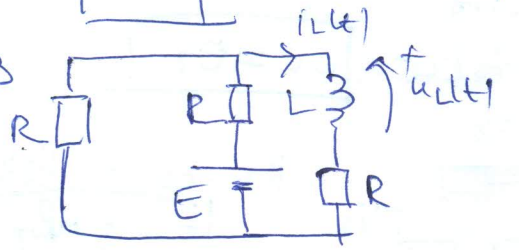
$U_{L0} = \phi$



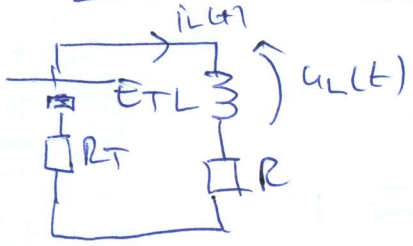
$E_T = RI = R\frac{E}{2R} = \frac{E}{2}$

$R_T = R \| R = R/2$

$t>0$



$\Rightarrow$



$$E_T - u_L(t) - R i_L(t) - R_T i_L(t) = 0$$

$$\frac{E}{2} - u_L(t) - \left(R + \frac{R}{2}\right) i_L(t) = 0$$

$$u_L(t) = L\frac{di_L(t)}{dt}$$

$$L\frac{di_L(t)}{dt} + \frac{3}{2}R i_L(t) = \frac{E}{2}$$

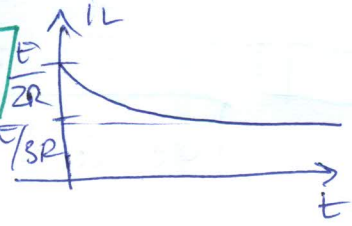
$$\frac{di_L(t)}{dt} + \frac{i_L(t)}{2L/3R} = \frac{E}{2L}$$

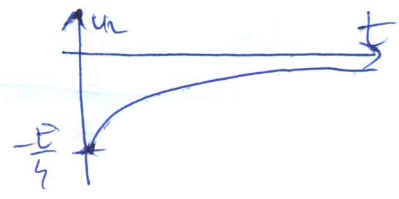
$$\tau = \frac{2L}{3R} \qquad K = \frac{E}{2L} \qquad B = \tau K = \frac{E}{3R}$$

$$A + B = I_{L0} \Rightarrow A = \frac{E}{2R} - \frac{E}{3R} = \frac{E}{6R}$$

$$i_L(t) = Ae^{-t/\tau} + B = \frac{E}{6R}e^{-t/\tau} + \frac{E}{3R}$$

$$u_L(t) = L\frac{di_L(t)}{dt} = -\frac{E}{4}e^{-t/\tau}$$





b) $E - u_R - u_L - R i_L = 0$

$$u_R(t) = E - u_L - R i_L = E + \frac{E}{4}e^{-\frac{3R}{2L}t} - \frac{E}{6}e^{-\frac{3R}{2L}t} - \frac{E}{3} = \frac{2E}{3} + \frac{E}{12}e^{-\frac{3R}{2L}t}$$

$$u_R(t)\big|_{t=2L/3R} = \frac{2E}{3} + \frac{E}{12}e^{-1}$$

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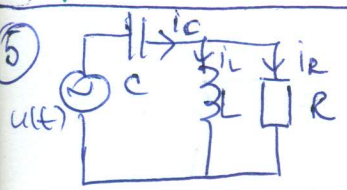
⑤



$$\bar{Z}_C = +\frac{1}{j\omega C} = -j5\,\Omega \qquad \bar{U} = \frac{20}{\sqrt{2}}e^{j0} = 10\sqrt{2}\,V$$

$$\bar{Z}_L = j\omega L = j5\,\Omega \qquad \bar{Z}_R = R = 5\,\Omega$$

$$\bar{Z}_{LR} = \frac{\bar{Z}_L \cdot \bar{Z}_R}{\bar{Z}_L + \bar{Z}_R} = \frac{5}{2}(1+j)\,\Omega$$

$$\bar{I}_C = \frac{\bar{U}}{\bar{Z}_C + \bar{Z}_{LR}} = \frac{10\sqrt{2}}{-j5 + \frac{5}{2} + j\frac{5}{2}} = \frac{10\sqrt{2}}{\frac{5}{2} - j\frac{5}{2}} = \frac{\frac{2}{5}10\sqrt{2}}{1-j} = \frac{4\sqrt{2}}{1-j} = 2\sqrt{2}(1+j)\,A = \bar{I}_C$$

$$\bar{U}_{LR} = \bar{Z}_{LR} \cdot \bar{I}_C = j10\sqrt{2}\,V$$

$$\bar{I}_L = \frac{\bar{U}_{LR}}{\bar{Z}_L} = 2\sqrt{2}\,A \qquad \bar{I}_R = \frac{\bar{U}_{LR}}{\bar{Z}_R} = j2\sqrt{2}\,A$$

b) $i_L(t) = 2\sqrt{2}\cdot\sqrt{2}\sin(\omega t + \theta) = 4\sin\omega t\,[A] = i_L(t)$

c) $\bar{S}_p = \bar{S}_u = \bar{U}\cdot\bar{I}_C^* = 40(1-j)\,VA$

$P_u = 40\,W$ $\qquad Q_u = -40\,VAr$ $\qquad S_u = 40\sqrt{2}\,VA$