Elektrotehnika

01. februar 2021.

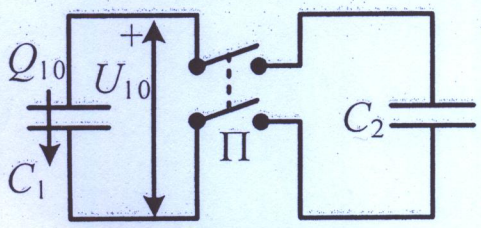
1. Sistem na Slici 1 čine: kondenzator kapaciteta C_1=8μF, koji je prethodno opterećen do napona U_{10}=100V, dvopolni prekidač Π u isključenom položaju i kondenzator kapaciteta C_2=4μF koji je neopterećen. Izračunati:

a) Ukupnu energiju sistema pri otvorenom prekidaču. **(4 poena)**

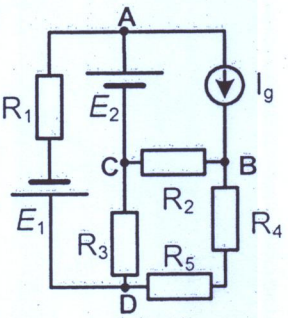
b) Napon na kondenzatorima i količine naelektrisanja na kondenzatorima nakon što se prekidač uključi, poveže kondenzatore i pošto se uspostavi stacionaro stanje. **(12 poena)**

c) Energiju sistema u ovom slučaju. **(4 poena)**

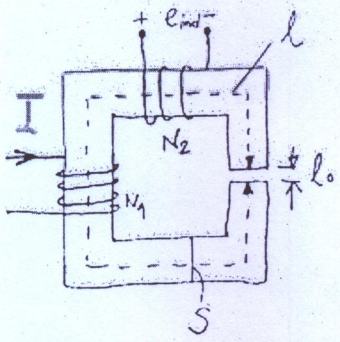
2. Primenom Tevenenove teoreme odrediti napon U_{BC} i snagu otpornika R_2 u kolu na Slici 2. Poznato je: E_1=10 V, E_2=40 V, R_1=R_3=R_4=R_5=10 Ω, R_2=25 Ω, $I_g = 1\text{A}$. **(20 poena)**



Slika 1



Slika 2



Slika 3

3. Na Slici 3 je prikazano magnetno kolo sa dva namotaja. Namotaj sa N_2 navojaka je otvorenih krajeva, a kroz namotaj sa N_1 navojaka protiče struja konstantnog intenziteta I. Jezgro je površine poprečnog preseka S, dužine srednje linije l i sadrži vazdušni procep debljine l_0. Magnetna permeabilnost jezgra iznosi μ.

a) Odrediti izraz za intenzitet vektora jačine magnetnog polja u jezgru. **(10 poena)**

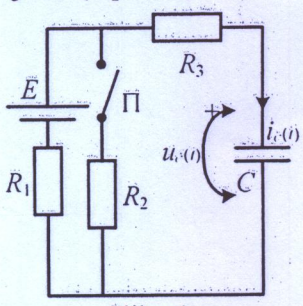
b) Odrediti izraz za induktivnost namotaja sa N_2 navojaka. **(5 poena)**

c) Odrediti elektromotornu silu indukovanu na krajevima namotaja sa N_2 navojaka. **(5 poena)**

4. U kolu na Slici 4 poznati su parametri elemenata: E, R_1=R, R_2=2R, R_3=3R i C. Prekidač Π je zatvoren i u kolu je uspostavljeno stacionarno stanje. U trenutku $t = 0$, prekidač se otvara.

a) Odrediti izraz za struju i napon kondenzatora nakon otvaranja prekidača **(12 p)** i nacrtati odgovarajuće vremenske dijagrame **(4 p)**.

b) Odrediti snagu generatora E u trenutku t_1=8RC. **(4 p)**



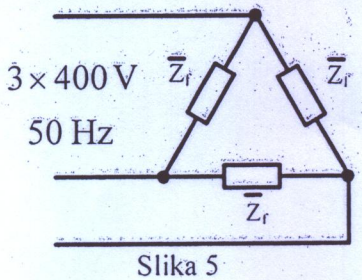
Slika 4

5. Kroz pretežno induktivni potrošač, koji je priključen na izvor naizmeničnog napona $U = 1000\,\text{V}$, $\omega = 500\,\text{rad/s}$, protiče struja efektivne vrednosti $I = 5\,\text{A}$. Aktivna snaga potrošača iznosi $P = 3\,\text{kW}$. Odrediti kapacitivnost kondenzatora koji treba priključiti paralelno potrošaču da bi se faktor snage podigao na vrednost $\cos\varphi = 0.8$. **(20 poena)**

6. Na Slici 5 prikazan je simetrični trofazni, pretežno kapacitivni, potrošač, priključen na trofazni sistem napona $3\times400\,\text{V}$. Poznata je aktivna snaga trofaznog potrošača $P = 6\sqrt{3}\,\text{kW}$ i efektivna vrednost linijske struje $I_l = 10\sqrt{3}\,\text{A}$. Odrediti:

a) Kompleksnu impedansu potrošača, $\underline{Z}_f$. **(15 poena)**

b) Faktor snage potrošača, $\cos\varphi$. **(5 poena)**



Slika 5

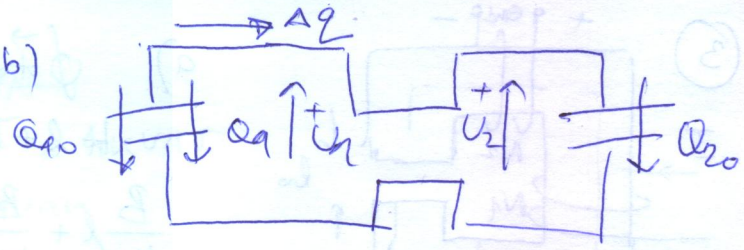
① a) $Q_{10} = C_1 U_{10} = 800\,\mu C$

$W_{10} = \frac{1}{2} Q_{10} U_{10} = 40\,\mu J$

$W_{20} = 0$

$\boxed{W_0 = W_{10} + W_{20} = 40\,\mu J}$

b)



b) $\left.\begin{aligned} Q_1 &= Q_{10} - \Delta q \\ Q_2 &= +\Delta q \\ U_1 = U_2 &= \frac{Q_1}{C_1} = \frac{Q_2}{C_2} \end{aligned}\right\} \Rightarrow \frac{Q_{10} - \Delta q}{C_1} = \frac{\Delta q}{C_2}$

$\Rightarrow \Delta q = \frac{Q_{10}}{1 + C_1/C_2} = \frac{800}{3}\,\mu C$

$\boxed{Q_1 = Q_{10} - \Delta q = \frac{1600}{3}\,\mu C}$

$\boxed{Q_2 = \Delta q = \frac{800}{3}\,\mu C}$

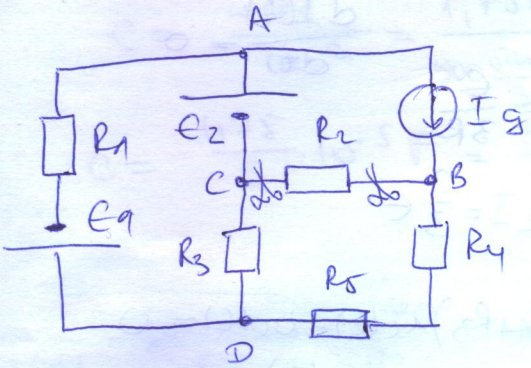
$\boxed{U_1 = U_2 = \frac{Q_2}{C_2} = \frac{200}{3}\,V}$

c) $W_{11} = \frac{1}{2} Q_1 U_1 = \frac{160}{9}\,\mu J$

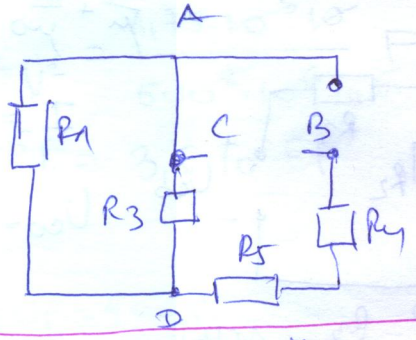
$W_{21} = \frac{1}{2} Q_2 U_2 = \frac{80}{9}\,\mu J$

$\boxed{W_1 = W_{11} + W_{21} = \frac{240}{9}\,\mu J = \frac{80}{3}\,\mu J}$

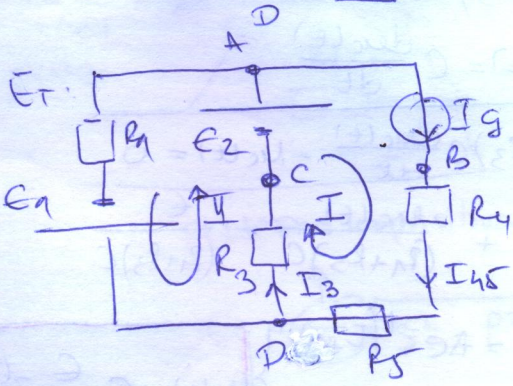
②



R_T



$\boxed{R_T = R_4 + R_5 + R_3 \| R_1 = 10 + 10 + 5 = 25\,\Omega}$



$\varepsilon_T = U_{BC}^{OV}$

$I_I = I_g = 1A$

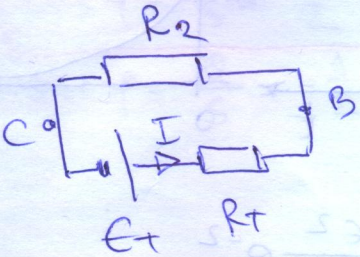
$R_3 I_I + (R_1 + R_3) I_{II} = \varepsilon_1 + \varepsilon_2$

$I_{II} = \frac{\varepsilon_1 + \varepsilon_2 - R_3 I_g}{R_1 + R_3} = 2A$

$I_{45} = I_I = 1A$

$I_3 = I_I + I_{II} = 3A$

$\boxed{\varepsilon_T = R_3 I_3 + (R_4 + R_5) I_{45} = 50V}$

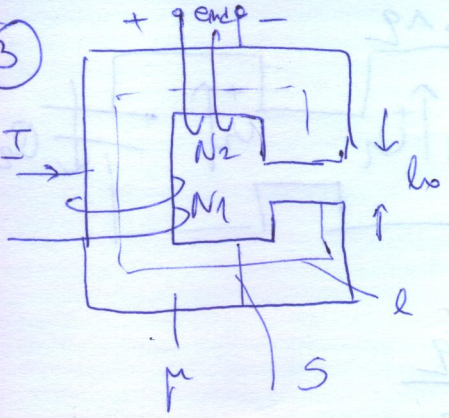


$I = \frac{\varepsilon_T}{R_T + R_2} = \frac{50}{25+25} = 1A$

$\boxed{U_{BC} = R_2 I = 25V}$

$\boxed{P_{R_2} = R_2 I^2 = 25W}$

③



a) $\oint \vec{H} d\vec{l} = \sum I$

$H \cdot l + H_0 l_0 = N_1 I$

$\frac{B}{\mu} l + \frac{B}{\mu_0} l_0 = N_1 I$

$B = \frac{N_1 I}{\frac{l}{\mu} + \frac{l_0}{\mu_0}}$

$\Phi = B \cdot S = B_0 S$

$\Rightarrow B = B_0$

$H = \frac{B}{\mu}$

$H_0 = \frac{B_0}{\mu_0} = \frac{B}{\mu_0}$

$H = \frac{B}{\mu} = \frac{N_1 I}{\mu\left(\frac{l}{\mu} + \frac{l_0}{\mu_0}\right)} \Rightarrow \boxed{H = \frac{N_1 I}{l + \frac{\mu}{\mu_0} l_0}}$

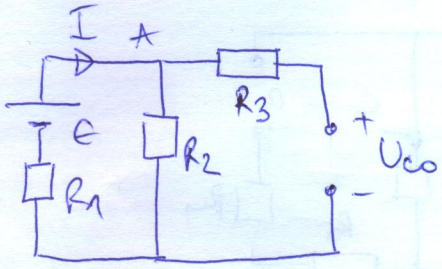
b) $\boxed{L_2 = \frac{N_2^2}{R_m + R_{m0}}} = \frac{N_2^2}{\frac{l}{\mu S} + \frac{l_0}{\mu_0 S}} = \boxed{\frac{N_2^2 S}{\frac{l}{\mu} + \frac{l_0}{\mu_0}}}$

c) $\boxed{e_{ind} = -N_2 \frac{d\Phi_S}{dt}} = = -\frac{N_1 N_2 S}{\frac{l}{\mu} + \frac{l_0}{\mu_0}} \frac{dI}{dt} = \boxed{0}$ jer je $I(t) = I = \text{const}$

$\frac{dI(t)}{dt} = 0$

④

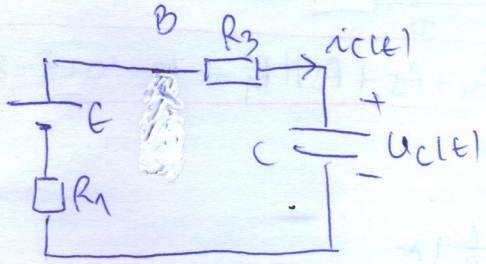
a) $\Pi \to \exists$



$I = \frac{\epsilon}{R_1 + R_2} = \frac{\epsilon}{3R}$

$U_{co} = U_{AB} = R_2 I = \frac{2}{3}\epsilon$

$\Pi \to \exists$



$\epsilon - (R_1 + R_3) i_c(t) - u_c(t) = 0$

$i_c(t) = C \frac{du_c(t)}{dt}$

$\epsilon - (R_1 + R_3) C \frac{du_c(t)}{dt} - u_c(t) = 0$

$\frac{du_c(t)}{dt} + \frac{u_c(t)}{(R_1 + R_3) C} = \frac{\epsilon}{(R_1 + R_3) C}$

$\tau = C(R_1 + R_3) = 4RC$

$K = \frac{\epsilon}{C(R_1 + R_3)} = \frac{\epsilon}{4RC}$

$u_c(t) = Ae^{-t/\tau} + B$

$B = K \cdot \tau = \epsilon$

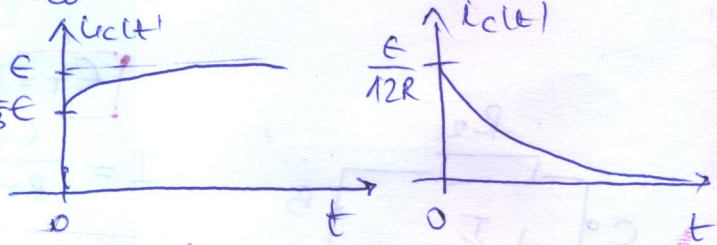
$A + B = U_{co}$

$\Rightarrow A = -\frac{\epsilon}{3}$

$\boxed{u_c(t) = \epsilon - \frac{\epsilon}{3} e^{-t/\tau}}$

$i_c(t) = C \frac{du_c(t)}{dt} = -\frac{\epsilon}{3}\left(-\frac{1}{\tau}\right) e^{-t/\tau}$

$= \frac{C\epsilon}{3} \cdot \frac{1}{4RC} e^{-t/\tau} = \boxed{\frac{\epsilon}{12R} e^{-t/\tau} = i_c(t)}$



b) $P_\epsilon(t) = i_\epsilon(t) \cdot \epsilon = i_c(t) \cdot \epsilon$

$P_\epsilon(t_1) = \epsilon \cdot i_c(t_1) = \frac{\epsilon^2}{12R} e^{-\frac{t_1}{\tau}} = \frac{\epsilon^2}{12R} e^{-\frac{8RC}{4RC}} = \frac{\epsilon^2}{12R} e^{-2}$

$\boxed{P_\epsilon(t_1) = \frac{\epsilon^2}{12R e^2}}$

(5) $U = 1000\,V$
$\omega = 500\,\frac{rad}{s}$
$I = 5A$

$P = 3kW$
$S = UI = 5kVA$
$Q = +\sqrt{S^2 - P^2} = 4kVAr$
↑ induktivan

Kada se veže kondenzator:

$P_{uk} = P + P_C^{\,\nearrow 0} = P = 3kW$

$\cos\varphi = 0{,}8 = \frac{8}{10} = \frac{4}{5}$

$S_{uk} = \frac{P_{uk}}{\cos\varphi} = \frac{3k}{0{,}8} = \frac{3k}{\frac{4}{5}} = \frac{15}{4}kVA = 3{,}75kVA$

$\sin\varphi = +\sqrt{1-\cos^2\varphi} = 0{,}6 = \frac{3}{5}$
↓ uzima se induktivno

$Q_{uk} = +\sqrt{S_{uk}^2 - P_{uk}^2}$ ← uzima se induktivno

$= \sqrt{\frac{15^2}{4^2} - 3^2}\,kVAr = \frac{\sqrt{225 - 4^2 \cdot 9}}{4}kVAr = \frac{\sqrt{225-16\cdot 9}}{4}kVAr$

$= \frac{\sqrt{81}}{4}kVAr = \frac{9}{4}kVAr = 2{,}25\,kVAr = Q + Q_C = Q - \omega C U^2$

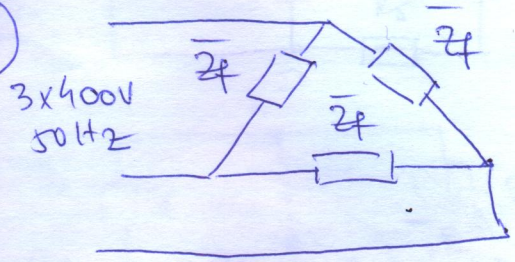
$Q_C = Q_{uk} - Q = -1{,}75\,kVAr$

$C = \frac{-Q_C}{\omega U^2} = \frac{1{,}75kVAr}{500\frac{rad}{s}\,(10^3)^2\,V^2} = \frac{1{,}75\cdot 10^3\,10}{5\cdot 10^2\cdot 10^6\,10^5}\,F$

$C = \frac{1{,}75}{5}\cdot 10^{-5}F = \frac{3{,}5}{10}\cdot 10^{-5}F = 3{,}5\cdot 10^{-6}F$

$\boxed{C = 3{,}5\mu F}$

(6)



$U_f = U_l = 400V$
$I_l = 10\sqrt{3}A$
$I_f = \frac{I_l}{\sqrt{3}} = 10A$

$S = 3U_f I_f = 12000VA = 12kVA$
$P = 6\sqrt{3}kW$

pretežno kapacitivan

$Q = -\sqrt{S^2 - P^2} = -\sqrt{12^2 - 6^2\cdot 3} = -\sqrt{6^2(4-3)} = -6kVAr$

$Z_f = \frac{U_f}{I_f} = \frac{400V}{10A} = 40\Omega$

$\boxed{\cos\varphi = \frac{P}{S} = \frac{\sqrt{3}}{2}}$

$\sin\varphi = \frac{Q}{S} = -\frac{1}{2}$

$\bar{Z}_f = Z_f\cos\varphi + j\,Z_f\sin\varphi$

$\bar{Z}_f = 40\left(\frac{\sqrt{3}}{2} - j\frac{1}{2}\right)$

$\boxed{\bar{Z}_f = (20\sqrt{3} - j20)\Omega}$