

PRIMER 1

$$u(t) = \overset{1)}{10} \cos(\overset{2)}{314}t + \overset{3)}{+\pi/4} \text{ V}$$

$$1) U_m = 10 \text{ V} \Rightarrow U = \frac{U_m}{\sqrt{2}} = \frac{10}{\sqrt{2}} = 5\sqrt{2} \text{ V}$$

$$2) \begin{cases} \omega = 314 \text{ rad/s} \\ f = \frac{\omega}{2\pi} \approx 50 \text{ Hz}, T = \frac{1}{f} = 20 \text{ ms} \end{cases}$$

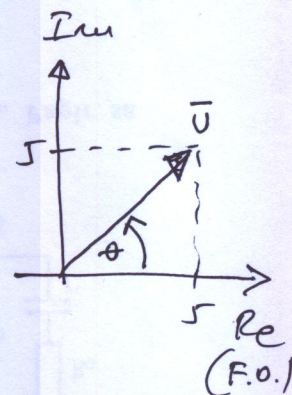
$$3) \theta = \pi/4$$

$$\bar{U} = U e^{j\theta}$$

$$\bar{U} = 5\sqrt{2} e^{j\pi/4} \text{ V}$$

$$\bar{U} = 5\sqrt{2} (\cos \pi/4 + j \sin \pi/4)$$

$$\bar{U} = 5 + j5 \text{ V}$$



PRIMER 2

$$\bar{U} = -5\sqrt{2} - j5\sqrt{2} \text{ V}$$

$$2) f = 100 \text{ Hz} \Rightarrow \omega = 2\pi f = 200\pi \frac{\text{rad}}{\text{s}} \\ \omega \approx 628 \frac{\text{rad}}{\text{s}}$$

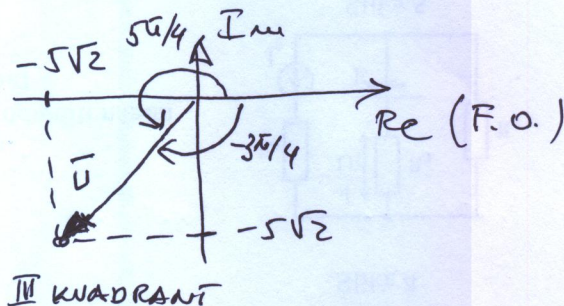
$$1) U = \sqrt{\text{Re}(\bar{U})^2 + \text{Im}(\bar{U})^2}$$

$$U = \sqrt{(-5\sqrt{2})^2 + (-5\sqrt{2})^2}$$

$$U = \sqrt{2 \cdot (5\sqrt{2})^2} = 5\sqrt{2} \sqrt{2} = 10 \text{ V}$$

$$U_m = U\sqrt{2} = 10\sqrt{2} \text{ V}$$

$$3) \theta = +5\pi/4 \text{ ili } \theta = -3\pi/4 \\ (\text{svejedno je?})$$



$$\Rightarrow \bar{U} = 10 e^{-j3\pi/4} \text{ V}$$

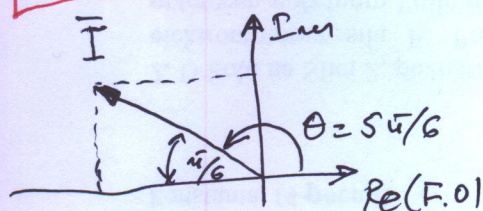
$$u(t) = U_m \cos(\omega t + \theta)$$

$$u(t) = \underbrace{10\sqrt{2}}_1 \cos(\underbrace{628t}_2 - \underbrace{3\pi/4}_3)$$

Kosinus po dogovoru

PRIMER 3

$$\bar{I} = -10\sqrt{3} + j10 \text{ A} = 20 \left(-\frac{\sqrt{3}}{2} + j\frac{1}{2} \right) = 20 e^{j5\pi/6}$$



$$\begin{aligned} & \uparrow \quad \uparrow \\ & -\cos(\pi/6) \quad \sin(\pi/6) = \sin(5\pi/6) \\ & = \cos(5\pi/6) \end{aligned}$$

$$I = \sqrt{(-10\sqrt{3})^2 + 10^2} = 10\sqrt{3+1} = 20 \text{ A}$$

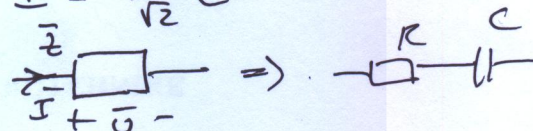
$$i(t) = I_m \cos(\omega t + \theta) = 20\sqrt{2} \cos(314t + 5\pi/6) \text{ A}$$

PRIMER 4

$$u(t) = 30 \cos(314t - 15^\circ) \Rightarrow \bar{U} = \frac{30}{\sqrt{2}} e^{-j\pi/12} \quad (15^\circ = \pi/12 \text{ rad})$$

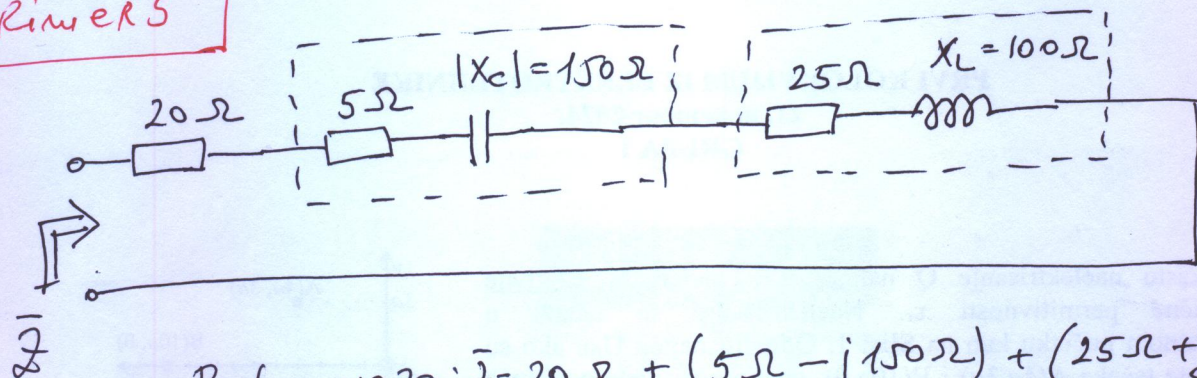
$$i(t) = 15 \cos(314t + \pi/12) \Rightarrow \bar{I} = \frac{15}{\sqrt{2}} e^{+j\pi/12}$$

$$\bar{Z} = \frac{\bar{U}}{\bar{I}} = \frac{30/\sqrt{2} e^{-j\pi/12}}{15/\sqrt{2} e^{+j\pi/12}} = 2 e^{-j\pi/6} \Omega$$



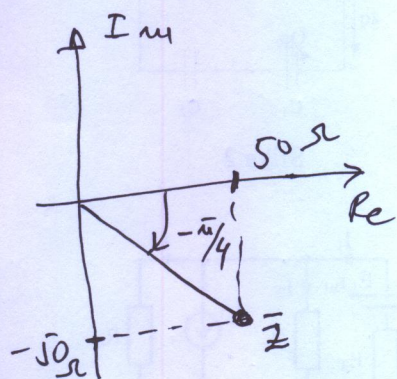
$$\bar{Z} = 2(\cos \pi/6 - j \sin \pi/6) = \sqrt{3} - j \Omega = R - j \frac{1}{\omega C} \Rightarrow R = \sqrt{3} \Omega, C = \frac{1}{314} \approx 3,18 \mu\text{F}$$

Primer 5



Redna veza: $\bar{Z} = 20\Omega + (5\Omega - j150\Omega) + (25\Omega + j100\Omega)$

$$\bar{Z} = \underset{(20+5+25)}{50\Omega} + j(100-150) = 50\Omega - j50\Omega$$



$$\bar{Z} = (50 - j50)\Omega = 50\sqrt{2} \cdot e^{-j\pi/4} \Omega \quad \text{KAPACITIVAN}$$

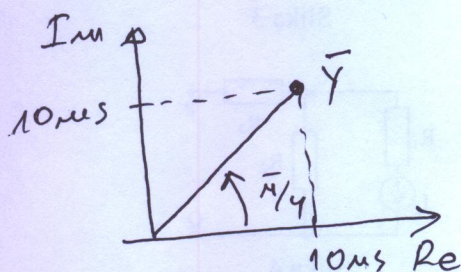
$$|\bar{Z}| = 50\sqrt{2} \Omega$$

$$\gamma = \frac{1}{Z} = \frac{1}{50\sqrt{2}} (1000 \text{ ms}) = \frac{20}{\sqrt{2}} \text{ ms}$$

$$\gamma = 10\sqrt{2} \text{ ms}$$

$$\bar{\gamma} = \frac{1}{\bar{Z}} = \frac{1}{50\sqrt{2} e^{-j\pi/4}} = 10\sqrt{2} e^{j\pi/4} \text{ ms}$$

$$\bar{\gamma} = (10 + j10) \text{ ms}$$



Primer 6

$$\left. \begin{array}{l} \bar{Z} = (20 + j10)\Omega \\ \bar{I} = (4 + j3)\text{A} \end{array} \right\} \Rightarrow \bar{U} = \bar{Z} \bar{I} = (20 + j10)(4 + j3)$$

$$\bar{U} = 80 + j40 + j60 - 30 = (50 + j100)\text{V}$$

$$\bar{S} = \bar{U} \bar{I}^* = (50 + j100)(4 + j3)^* = (50 + j100)(4 - j3) = 200 + j400 - j150 + 300$$

$$\bar{S} = (500 + j250) \text{ VA} \Rightarrow P = 500 \text{ W}, Q = 250 \text{ VAR}$$

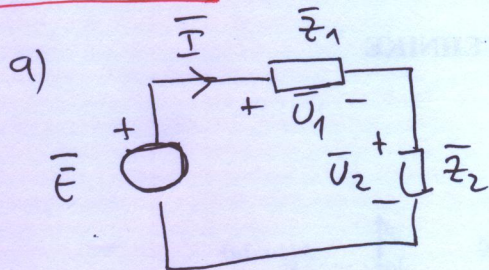
$$\text{ili} \quad \bar{S} = \bar{Z} I^2 = (20 + j10) \cdot (\sqrt{4^2 + 3^2})^2 = (20 + j10) \cdot 25 = (500 + j250) \text{ VA} \quad \checkmark \text{ oigledno OK}$$

$$\text{ili} \quad \left. \begin{array}{l} P = RI^2 = 20 \cdot (\sqrt{4^2 + 3^2})^2 = 20 \cdot 25 = 500 \text{ W} \\ Q = XI^2 = 10 \cdot (\sqrt{4^2 + 3^2})^2 = 10 \cdot 25 = 250 \text{ VAR} \end{array} \right\} \bar{S} = (500 + j250) \text{ VA}$$

$$S = |\bar{S}| = \sqrt{P^2 + Q^2} = \sqrt{500^2 + 250^2} = 250\sqrt{2^2 + 1^2} = 250\sqrt{5} \text{ VA}$$

$$\text{ili} \quad S = UI = \sqrt{50^2 + 100^2} \cdot \sqrt{4^2 + 3^2} = 50\sqrt{5} \cdot 5 = 250\sqrt{5} \text{ VA} \quad \text{ili} \quad S = Z I^2 = 10\sqrt{5} \cdot 25 = 250\sqrt{5} \text{ VA}$$

Primer 7



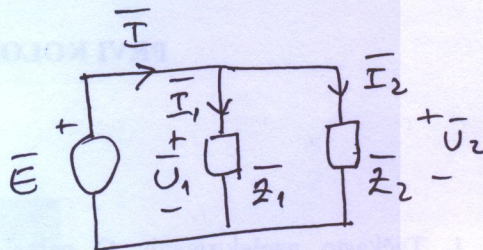
$$\bar{I} = \frac{\bar{E}}{\bar{Z}_1 + \bar{Z}_2} = \frac{\bar{E}}{R_1 + jX_1 + R_2 + jX_2}$$

$$\bar{I} = \frac{\bar{E}}{(R_1 + R_2) + j(X_1 + X_2)}$$

$$\bar{U}_1 = \bar{Z}_1 \bar{I}_1 = \dots$$

$$\bar{U}_2 = \bar{Z}_2 \bar{I}_2 = \dots$$

b)



$$\bar{U}_1 = \bar{U}_2 = \bar{E}$$

$$\bar{I}_1 = \frac{\bar{U}_1}{\bar{Z}_1} = \frac{\bar{E}}{R_1 + jX_1}$$

$$\bar{I}_2 = \frac{\bar{U}_2}{\bar{Z}_2} = \frac{\bar{E}}{R_2 + jX_2}$$

$$\bar{I} = \bar{I}_1 + \bar{I}_2 = \dots$$

Primer 8

$$\bar{Z} = (3 + j4) \Omega \quad U = 10 \text{ V}, \quad \omega = 1 \text{ rad/s}$$

$$Z = |\bar{Z}| = \sqrt{3^2 + 4^2} = 5 \Omega$$

$$I = \frac{U}{Z} = \frac{10}{5} = 2 \text{ A}$$

$$P = R \cdot I^2 = 3 \cdot 2^2 = 12 \text{ W}$$

$$Q = X I^2 = 4 \cdot 2^2 = 16 \text{ VAR}$$

$$S = Z I^2 = 5 \cdot 2^2 = 20 \text{ VA}$$

$$\cos \varphi = \frac{R}{Z} = \frac{3}{5} = 0,6$$

$$\text{ili} \quad \cos \varphi = \frac{P}{S} = \frac{12}{20} = 0,6$$

$$\cos \varphi_k = 1 \Rightarrow Q_{vk} = Q_c + Q = 0$$

$$Q_c = -Q$$

$$- \omega C U^2 = -16 \text{ VAR} \quad (-Q)$$

$$C = \frac{Q}{\omega U^2}$$

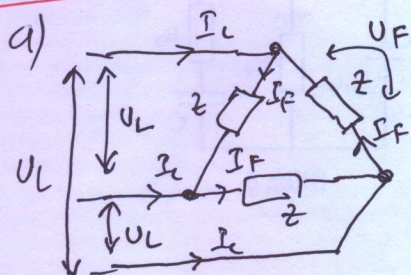
$$C = \frac{16}{1 \cdot 10^2} = \frac{16}{100} = 160 \mu\text{F}$$

$$\text{ili} \quad X_L = \omega L \Rightarrow 4 = 1 \cdot L \Rightarrow L = 4 \text{ H}$$

$$C = \frac{L}{R^2 + (\omega L)^2} = \frac{4}{3^2 + 4^2} = \frac{4}{25} = 160 \mu\text{F}$$

Primer 9

$$Z = \sqrt{3^2 + 4^2} = 5 \Omega$$



$$3 \times 100 \text{ V} \Rightarrow U_L = 100 \text{ V}$$

$$U_F = U_L = 100 \text{ V}$$

$$I_F = \frac{U_F}{Z} = \frac{100}{5} = 20 \text{ A}$$

$$I_L = \sqrt{3} I_F = 20\sqrt{3} \text{ A}$$

$$\bar{S} = 3 \bar{Z} I_F^2$$

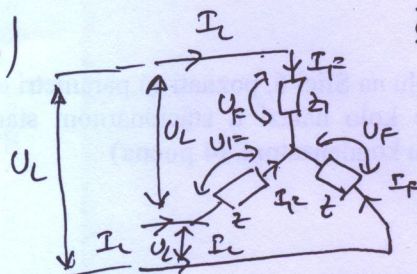
$$\bar{S} = 3 \cdot (3 + j4) 20^2$$

$$\bar{S} = (3600 + j4800) \text{ VA}$$

$$P = 3600 \text{ W}$$

$$Q = 4800 \text{ VAR}$$

b)



$$3 \times 100 \text{ V} \Rightarrow U_L = 100 \text{ V}$$

$$U_F = \frac{U_L}{\sqrt{3}} = \frac{100}{\sqrt{3}} = \frac{100\sqrt{3}}{3} \text{ V}$$

$$I_F = \frac{U_F}{Z} = \frac{100\sqrt{3}/3}{5} = \frac{20}{3} \sqrt{3} \text{ A}$$

$$I_L = I_F = \frac{20}{3} \sqrt{3} \text{ A}$$

$$\bar{S} = 3 \bar{Z} I_F^2$$

$$\bar{S} = 3(3 + j4) \frac{20^2}{3^2} \cdot 3$$

$$\bar{S} = (1200 + j1600) \text{ VA}$$

$$P = 1200 \text{ W}$$

$$Q = 1600 \text{ VAR}$$