MODELS BASED ON RISK EQUITY

On the concept of equity

Gopalan R. et al. "Modeling equity of risk in the transportation of hazardous materials", Operations Research Vol. 38, No. 6, November-December 1990

- Equity as a concept in public services is not new. For example, Morrell (1984) discusses an equitable solution to the problem of citing HM dump sites. ***His solution is to cite numerous dump sites in various counties simultaneously, with capacities in proportion to each county's waste generation***. Thus, no community feels singled out for having more than their fair share of dump sites. Morrell also presents an excellent discussion on the politics of equity and advocates equity as a meaningful concept in facility location.

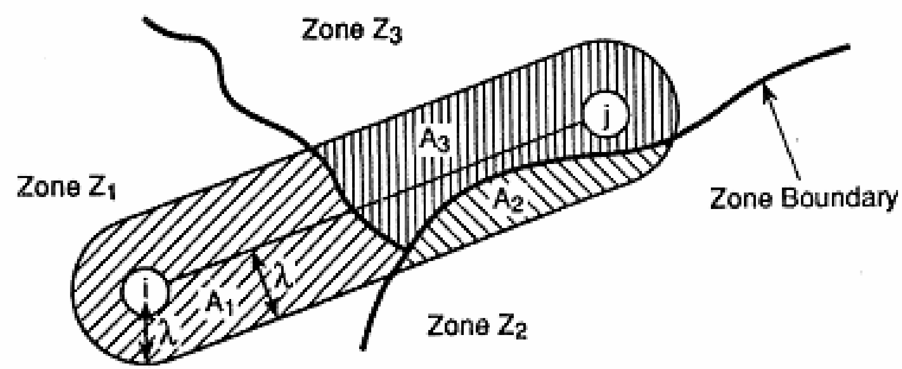
MORRELL, D.1984. Siting and the Politics of Equity. Hazardous Waste, 555-571.

- Another example of work based on equity in public service systems are the papers by Keeney (1980a,b), in which he ***expresses equity as the magnitude of the largest difference in the level of risk among a fixed set of individuals***. Specifically, Keeney holds total risk constant and compares different distributions of that constant total risk across individuals. This is in contrast to the formulation we present in this paper, as we examine an explicit tradeoff between global risk and equity in the distribution of that risk.

KEENEY, R. L. 1980a. Equity and Public Risk. Opns. Res. 28, 527-534.

KEENEY, R. L. 1980b. Utility Functions for Equity and Public Risk. Mgmt. Sci., 345-353.

- While it is relatively easy to formulate a travel-time matrix for a given network, it is not as straightforward to calculate the risk of travel between pairs of nodes. Risk estimation depends, in part, upon individual perceptions of risk and therefore tends to exhibit a high degree of variance.
- By far the most popular approach to estimate risk is to multiply the probability of an accident with estimated consequence to evaluate expected damage. The estimated consequence is often measured as potential fatalities or the dollar damage to property, etc.
- One widely used assumption that aids this estimation is that, in the event of an accident and container rupture, the HM has a radius of spread that depends on factors like the physical and chemical properties of the substance in question. If X represents the radius of spread, population who live within the boundary of a circle with a radius X and a center at the scene of the accident could potentially be affected. For travel on a link, we could speak of a whole X-neighborhood that is endangered.



- The X-neighborhood is a concept developed by Batta and Chiu (1988), and is useful for the purpose of collecting data for our model, as we will detail.
- Consider, for example, a link (i, j) whose X-neighborhood could potentially endanger three zones of a geographical region, as shown in Figure. By using well documented analytically or empirically derived risk analysis models, we define a risk function f(x,y) over the X-neighborhood (after describing it in a Cartesian coordinate system). Provided that an integrable function f(x, y) is arrived at, we can compute the risk to a zone as

$$\pi_{Z_k}(i, j) = \text{Risk to zone } Z_k \text{ by travel on } (i, j)$$

$$= \int_{A_k}\int f(x, y)\, dx\, dy$$

for $k = 1, 2, 3$. The global risk of travel for travel on link (i, j), c_{ij}, can be viewed as

$$c_{ij} = \pi_{Z_1}(i, j) + \pi_{Z_2}(i, j) + \pi_{Z_3}(i, j).$$

Equity of risk – Integer programming formulation

- For our model, we are supplied with a planar, undirected transportation network defined by a node set N, INI = n, and an arc set A. An Origin (0) and a Destination (D) are defined for the obnoxious vehicle.
- The geographical region encompassed by the transportation network is assumed to be divided into K mutually disjoint zones. For notational convenience, we define a zone pair ordering as an ordered pair (Z_a, Z_b) of zones. We need to distinguish (Z_a, Z_b) from (Z_b, Z_a).
- Let T denote the maximum number of trips by which time we would like to achieve equity. We note that T may be far less than the total number of trips to be made. The objective is to find a set of T (not necessarily distinct) simple paths to minimize the total risk over the T trips while simultaneously keeping the difference in total risk between every zone pair within a threshold (Tμ), where the equity parameter A is the average equity over each trip. The restriction to a simple path is necessary to avoid pathological solutions which loop the vehicle in a neighborhood to give nearby areas more risk. Let x_{ijt}, be a binary decision variable, equal to 1 if link (i, j) is used on the t-th trip, and equal to 0 otherwise. A formulation for the multiple-trip problem is to

$$\text{minimize} \sum_{t=1}^{T} \sum_{(i,j)\in A} c_{ij} x_{ijt}$$

subject to

$$\sum_{t=1}^{T} \sum_{(i,j)\in A} (\Pi_{Z_a}(i,j) - \Pi_{Z_b}(i,j))x_{ijt} \leqslant T\mu$$

$$\text{for all } a, b = 1, \ldots, K \quad (1)$$

$$\sum_j x_{ijt} - \sum_j x_{jit} = \begin{cases} 1 & \text{if } i = \mathrm{O} \\ -1 & \text{if } i = \mathrm{D} \\ 0 & \text{otherwise} \end{cases}$$

$$\text{for all } i = 1, \ldots, n \quad \text{and for all } t = 1, \ldots, T \quad (2)$$

$$\sum_j x_{ijt} \leqslant 1$$

$$\text{for all } i = 1, \ldots, n \quad \text{and for all } t = 1, \ldots, T \quad (3)$$

$$x_{ijt} \in \{0, 1\}$$

$$\text{for all } i, j = 1, \ldots, n \quad \text{and for all } t = 1, \ldots, T. \quad (4)$$

Solution proposed is by Lagrangean relaxation

Constraint set (1) has to be repeated for every zone pair ordering (Z_a, Z_b) rather than just some selection of a pair of zones. This is because we do not know a priori which pair of zones is going to sustain the greatest difference in risk for travel on an arbitrary path. Constraint sets (2) and (4) together ensure that the solutions describe paths from origin to destination. Finally, constraint set (3) restricts attention to simple paths.

Note that the equity constraint (1) is written as the difference in allocated risk between two zones, summed over links and trips.

Risk equity in facility location and transportation of hazardous materials

Current J., Ratick S. (1995) "A model to assess risk, equity and efficiency in facility location and transportation of hazardous materials", Location Science. Vol. 3. No. 3, pp. 187-201.

- In recent years there has been increased public and governmental concern regarding hazardous materials (HAZMAT) management as the magnitudes of such materials have increased. This concern has led to research on mathematical modeling approaches to aid decision makers in analyzing HAZMAT logistics decisions.
- Two important components of these decisions are the location of the HAZMAT facilities and the routing of the HAZMAT to and from these facilities.
- Facility location decisions and routing decisions are often based on multiple criteria. This is particularly true in HAZMAT logistics decisions. Cost minimization is an obvious consideration in these decisions. Due to the dangers involved in HAZMAT transport and treatment, risk minimization also plays an important role in HAZMAT logistics. Facility location and route selection decisions are spatial in nature, the resulting risks imposed are distributed spatially. Consequently, the equity of the distribution of these risks is also an important consideration in these decisions.
- Here is presented a multi objective approach to assist decision makers in analyzing combined location/routing decisions involving hazardous materials. The model includes objectives related to risk, equity and cost.
- The model presented here, ***REEM, includes five objectives related to risk, equity, and efficiency***.

- There are two objectives associated with risk.
    - One minimizes the total risk associated with transportation of HAZMAT and
    - the other minimizes the total risk associated with HAZMAT facility location.
- Similarly, there are two equity objectives.
    - One minimizes the maximum exposure to transport risk by any individual and
    - the other minimizes the maximum facility risk faced by any individual.
- These "minimax" objectives consider equity in a manner to improve the condition of those worst-off.
- The fifth objective addresses efficiency by minimizing the total transportation and facility costs.
- *Given the sources and quantities of the HAZMAT, REEM determines the locations of treatment or storage facilities and the transportation routes and quantities shipped from the sources to these facilities*.
- In general, these five objectives will be in conflict, i.e. no single solution will be optimal for all of them. For example, a solution that minimizes cost will ship large quantities over the least cost routes. This will expose the people along these routes to greater risk than those not along them. Optimizing the transportation equity objective on the other hand will lead to the transportation of smaller quantities over a larger number of routes, which will tend to increase transportation costs and the total risk involved but reduce the maximum exposure faced by anyone along the routes.

- The tradeoffs among these objectives are complex and the number of alternative location-routing options is large. Multi objective programming provides a method to identify efficient (i.e. non-inferior) solutions and the tradeoffs involved.

The model formulation

- Given a network G = (N,A) consisting of a set N of n nodes, a set A of m directed arcs (i,j) connecting node i to node j, and two non-negative weights a_{ij} and c_{ij} associated with each arc (i, j)∈A, REEM may be formulated as a mixed integer program as follows:

$$\text{Minimize} \sum_i \sum_j a_{ij} X_{ij} \tag{1}$$

$$\text{Minimize} \sum_{j \in F} a_i \left(\sum_i X_{ij} \right) \tag{2}$$

$$\text{Minimize } M \tag{3}$$

$$\text{Minimize } P \tag{4}$$

$$\text{Minimize} \sum_i \sum_j c_{ij} X_{ij} + \sum_j \left(f_j Y_j + h_j \sum_i X_{ij} \right) \tag{5}$$

subject to

$$\sum_{j} X_{ij} = w_i \quad \forall i \in S \tag{6}$$

$$\sum_{i} X_{ij} \leqslant k_j Y_j \quad \forall j \in F \tag{7}$$

$$\sum_{i} X_{il} - \sum_{j} X_{lj} = 0 \quad \forall l \notin S \text{ or } F \tag{8}$$

$$\sum_{i} X_{ij} \leqslant M \quad \forall j \notin S \text{ or } F \tag{9}$$

$$\sum_{i} X_{ij} \leqslant P \quad \forall j \in F \tag{10}$$

$$X_{ij} \geqslant 0 \quad \forall (i, j) \in A \tag{11}$$

$$Y_j \in \{0, 1\} \quad \forall j \in F \tag{12}$$

where

X_{ij} = the amount of waste shipped from node i to node j
Y_j = 1 if the facility is opened at j, 0 otherwise
w_i = the total amount of waste generated at i ($i \in S$)
a_{ij} = the total population within D units of arc (i, j)
a_j = the population density factor around facility site j ($j \in F$),
c_{ij} = per unit (e.g. truckload) cost of traversing arc (i, j)
f_j = the fixed cost of opening a facility at j
h_j = the per unit variable cost of operating a facility at j
k_j = the maximum capacity of a facility at j
F = the set of potential disposal facilities
S = the set of waste sources

- Constraint set (6) ensures that all waste at the waste sources (set $S \subset N$) is shipped from the sites.
- Constraint set (7) prohibits the shipping of waste to a facility if the facility, say j, is not opened (i.e. if $Y_j = 0$) and restricts shipments to an opened facility to be no more than the facility's capacity, k_j.
- Constraint set (8) requires all waste shipped into a transportation node to be shipped out of it if the node does not represent a potential facility site.

- Constraint set (9) defines M, the maximum amount shipped past any individual on the network. Note, this constraint is written for each node, not each arc. This is possible because all flow on an arc, say (i, j), must pass the tail node, j. Therefore, the maximum individual exposure will occur at a node as more than one arc may end at a node.
- Constraint set (10) defines the maximum quantity, P, that any individual is exposed to at a facility site.
- Constraint set (11) ensures non negative shipments.
- We assume that the shipments need not be integer. If not, we assume that non-integer shipments can be combined into integer shipments over time. This assumption greatly reduces the number of integer variables in the problem.
- Finally, constraint (12) enforces the binary nature of the sitting decision.
- Objectives (1) and (2) are the risk objectives where (1) minimizes the total transportation risk and (2) minimizes the total facility risk. Objectives (3) and (4) are the equity objectives which minimize the maximum transportation exposure faced by any individual (3) and the maximum facility risk faced by any individual (4). Objective (5) minimizes the total transportation, facility and operating costs of the system.
- The formulation of REEM is compact. There are 2n - |s| constraints, m continuous variables and |F| zero-one variables.
- Given that the number of potential disposal facility sites for hazardous materials, IFI, is generally not large, it should be possible to solve REEM using standard branch-and-bound binary integer approaches for most problem instances.

- The calculation of coefficient a_j in objective (2) requires some explanation. We have assumed that the radius of risk, r_j, at a facility is a function of the total quantity processed at the facility and takes the form $r_j = (\sum_i X_{ij})^{\frac{1}{2}}$. Consequently, the area at risk at facility j is $\pi \cdot r_j^2$.
- Given the population density, $\bar{a}_j$, around facility site j, the total exposed population at site j then equals $\bar{a}_j \pi \cdot r_j^2 = \bar{a}_j \pi \cdot \sum_i x_{ij}$. Setting $a_j = \bar{a}_j \pi$, we obtain (2).
- It should be noted that this formulation assumes that wastes cannot be transported through a generating node or a facility node. This was done for ease of exposition and these assumptions can be readily dropped by modifying the network as follows. For each $i \in S$, $j \in F$ which may serve as a transport node, create a dummy node i or j and a dummy arc (i, i') or (j, j') with $c_{ii'} = c_{jj'} = a_{ii'} = a_{jj'} = 0$ and add to G = (N, A). Replace i with i' in S and j with j' in F and solve [REEM] on the augmented network.